Quantitative Genetics in Multi-Objective Optimization Algorithms: From Useful Insights to Effective Methods

Roberto Santana
Computational Intelligence Group
Universidad Politécnica de Madrid, Spain
roberto.santana@upm.es

Hossein Karshenas
Computational Intelligence Group
Universidad Politécnica de Madrid, Spain
hkarshenas@fi.upm.es

Concha Bielza
Computational Intelligence Group
DIA, Universidad Politécnica de Madrid, Spain
mcbielza@fi.upm.es

Pedro Larrañaga
Computational Intelligence Group
DIA, Universidad Politécnica de Madrid, Spain
pedro.larranaga@fi.upm.es

ABSTRACT

This paper shows that statistical algorithms proposed for the quantitative trait loci (QTL) mapping problem, and the equation of the multivariate response to selection can be of application in multi-objective optimization. We introduce the conditional dominance relationships between the objectives and propose the use of results from QTL analysis and G-matrix theory to the analysis of multi-objective evolutionary algorithms (MOEAs).

Categories and Subject Descriptors

G.1 [**Optimization**]: Global optimization; G.3 [**Probabilistic methods**]

General Terms

Algorithms

Keywords

multi-objective optimization, fitness function, quantitative genetics, selection, probabilistic models

1. INTRODUCTION

One of the research fields from which nature-inspired ideas applied to evolutionary computation originated is quantitave genetics (the study of inheritance at the phenotypic level) [3]. An evolutionary view of phenotypic fitness traits points to the importance of selection for phenotypic diversification and to the fact that the correlated evolution of multiple traits is common [1]. Since the selection of single traits may be influenced by their correlations to other traits, methods from quantitative genetics try to incorporate these correlations in the theoretical framework for evolutionary analysis. In particular, the multivariate response to selection equation links the fitness gain of different traits to the G-matrix (the trait covariance matrix).

In our approach to methods from quantitative genetics, we are not only looking for metaphors that could be incorporated into the conception of more efficient algorithms. We are also in the search for conceptual frameworks that could increase the understanding of the way MOEAs behave. We recognize modeling as an essential element to expand the applicability of these algorithms to practical problems. Therefore, results from genetics that provide insight or theoretical tools for the conception, use, and interpretation of MOEA models are particularly valuable. The paper introduces the concepts of conditional dominance and conditional conflicting criteria and discusses the use G-matrix theory to the analysis of MOEAs.

GECCO'11, July 12–16, 2011, Dublin, Ireland.
ACM 978-1-4503-0690-4/11/07.

2. MODELING OF CONDITIONAL RELATIONSHIPS BETWEEN OBJECTIVES

One of the main uses of variable-objective mapping is to analyze the way in which objective relationships may be mediated by some common variables. The influence of the variable may be so critical, that for a given (fixed) variable's value, the type of relationship between the objectives may be of one type (e.g. conflicting objectives) and for a different variable value the type of relationship may change.

In this section, we extend the definitions of dominance and conflicting objectives introduced in [2] to capture the notion of dependence between objective relationships with respect to variables.

We consider a maximization problem with k objective functions $\mathbf{f}_i \ : \mathbf{X} \rightarrow \mathcal{R}, i = 1, \ldots k$, where the vector function $\mathbf{f}_i \ := (f_1, \ldots, f_k)$ maps each solution $\mathbf{x}$ to an objective vector $f(\mathbf{x}) \in \mathcal{R}^k$. It is also assumed that the underlying dominance structure is given by the weak Pareto dominance relation which is defined as follows:

$\preceq_{\mathcal{F}'} := \{(\mathbf{x}, \mathbf{y}) \mid \mathbf{x}, \mathbf{y} \in \mathbf{X} \wedge \forall f_i \in \mathcal{F}' \ : f_i(\mathbf{x}) \leq f_i(\mathbf{y})\}$, where $\mathcal{F}'$ is a set of objectives with $\mathcal{F}' \subseteq \mathcal{F} \ := \{f_1, \ldots, f_k\}$. The Pareto (optimal) set is given as $\{\mathbf{x} \in \mathbf{X} \mid \nexists \mathbf{y} \in \mathbf{X} \setminus \{\mathbf{x}\} : \mathbf{x} \preceq_{\mathcal{F}} \mathbf{y} \wedge \mathbf{y} \not\preceq_{\mathcal{F}} \mathbf{x}\}$.

We define a general dominance relation for a subset of solutions $\mathbf{X}_S, S \subset \{1,\ldots,n\}$, given by fixed values for some variables.

The *conditional weak Pareto dominance relation* is defined as follows:
$\preceq_{\mathcal{F}'}^{x'_S} := \{(\mathbf{x},\mathbf{y}) \mid \mathbf{x},\mathbf{y} \in \mathbf{X}, x_S = y_S = x'_S \wedge \forall f_i \in \mathcal{F}' : f_i(\mathbf{x}) \leq f_i(\mathbf{y})\}$.

Notice that for $S = \emptyset$, $\preceq_{\mathcal{F}'}^{x'_S} = \preceq_{\mathcal{F}'}$.

Definition 1. *Let $\mathcal{F}_1, \mathcal{F}_2 \subseteq \mathcal{F}$ be two sets of objectives. We say that*

- *$\mathcal{F}_1$ and $\mathcal{F}_2$ are not conditionally conflicting given $\mathbf{X}_S = x'_S$ iff $\preceq_{\mathcal{F}_1}^{x'_S} \subseteq \preceq_{\mathcal{F}_2}^{x'_S} \wedge \preceq_{\mathcal{F}_2}^{x'_S} \subseteq \preceq_{\mathcal{F}_1}^{x'_S}$.*
- *$\mathcal{F}_1$ and $\mathcal{F}_2$ are conditionally weakly conflicting given $\mathbf{X}_S = x'_S$ iff ($\preceq_{\mathcal{F}_1}^{x'_S} \subseteq \preceq_{\mathcal{F}_2}^{x'_S} \wedge \preceq_{\mathcal{F}_2}^{x'_S} \nsubseteq \preceq_{\mathcal{F}_1}^{x'_S}$) or ($\preceq_{\mathcal{F}_2}^{x'_S} \subseteq \preceq_{\mathcal{F}_1}^{x'_S} \wedge \preceq_{\mathcal{F}_1}^{x'_S} \nsubseteq \preceq_{\mathcal{F}_2}^{x'_S}$).*
- *$\mathcal{F}_1$ conditionally strongly conflicting with $\mathcal{F}_2$ given $\mathbf{X}_S = x'_S$ iff $\preceq_{\mathcal{F}_1}^{x'_S} \nsubseteq \preceq_{\mathcal{F}_2}^{x'_S} \wedge \preceq_{\mathcal{F}_2}^{x'_S} \nsubseteq \preceq_{\mathcal{F}_1}^{x'_S}$.*

The relationships between the problem criteria in different subspaces of the search space may change. Fixing some of the problem variables is only one of the possible ways to define subspaces of the search space. However, we argue that this type of decomposition is particularly suitable for a better understanding of multi-objective problems (MOPs). Conditional relationships between criteria given variables could be useful for a decision maker (DM) in the process of selecting the final solutions from the Pareto set approximation. It would be possible to identify problem variables that play a critical role in the way the objectives are related.

3. G-MATRIX IN MULTI-OBJECTIVE OPTIMIZATION ALGORITHMS

When selection in individual traits may produce changes in other related traits, the *response to selection on multiple traits* [1] or multivariate response to selection is used to predict the evolutionary selection acting on multiple traits. This concept is of direct application to the analysis of MOEAs because selection methods, as applied in MOEAs, take into consideration the information about all the objectives. A fundamental question is how the applied selection will influence the improvement in fitness values for all the objectives.

$$\Delta\mathbf{z} = G\beta \tag{1}$$

The multivariate breeder Equation (1) can be seen as a generalization of the response to selection to the case of multiple traits. It describes the vector of changes in mean trait values ($\Delta\mathbf{z}$) as a result of selection. β is a vector of selection gradients for the traits and G is the covariance between traits, i.e. G_{ii} represents the additive genetic variance of trait i and G_{ij} the covariance between traits i and j.

We have not found previous reports on the application of the G-matrix in the analysis of MOEAs. In [5], concepts from quantitative genetics were applied to the analysis of single-objective evolutionary algorithms. The multivariate response to selection equation and the G-matrix could be applied to MOEAs in the following general ways: 1) Predictions of MOEA behavior. 2) Reconstruction of the form of selection that has led to divergence among populations. 3) Comparison between MOEAs.

For a given selection method and reproduction operator, we can compute $\Delta\mathbf{z}$ from the mean objective values achieved in generations t and $t+1$. Similarly, β and G can be computed from the information available from the MOEA. We can use Equation (1) to estimate the agreement between $\Delta\mathbf{z}$ and $G\beta$. Investigating the G-matrix for different selection and reproduction methods is useful to understand the different dynamics that govern the behavior of MOEAs and how these dynamics are related to the existence of correlations between objectives.

4. CONCLUSIONS

The main contributions of this paper are: 1) To introduce the concepts of conditional dominance and conditional conflicting criteria and 2) To propose the use of the multivariate response to selection equation for the analysis of MOPs.

Statistical methods for automatic computation of the relationships between variables and objectives [4] open a new direction for MOEA enhancement. These methods can simultaneously improve the search efficiency and uncover the structure of the interactions in MOPs. Theoretical frameworks such as the multivariate response to selection allows us to compare different selection methods in terms of the selection differential they produce, but also in terms of their effect on the objectives covariance in subsequent populations. In the future, we expect the incorporation of this theoretical framework for the analysis, and potential improvement, of MOEAs.

5. ACKNOWLEDGMENTS

This work has been partially supported by the TIN2010-20900-C04-04, Consolider Ingenio 2010 - CSD2007-00018 projects (Spanish Ministry of Science and Innovation) and the CajalBlueBrain project.

6. REFERENCES

[1] S. Arnold. *Quantitative Genetic Studies of Behavioral Evolution*, chapter Multivariate inheritance and evolution: a review of concepts, pages 17–48. Chicago: University of Chicago Press, 1994.

[2] D. Brockhoff and E. Zitzler. Dimensionality reduction in multiobjective optimization: The minimum objective subset problem. In K.-H. Waldmann and U. M. Stocker, editors, *Operation Research Proceedings 2006. Selected Papers of the Annual International Conference of the German Operations Research Society*, pages 423–429, 2006.

[3] D. S. Falconer and T. F. C. Mackay. *Introduction to Quantitative Genetics*. Addison Wesley Longman, Harlow, Essex, UK, 1996.

[4] H. Karshenas, R. Santana, C. Bielza, and P. Larrañaga. Multi-objective optimization with joint probabilistic modeling of objectives and variables. In *Evolutionary Multi-Criterion Optimization: Sixth International Conference, EMO 2011*, volume 6576 of *Lecture Notes in Computer Science*, pages 298–312. Springer Berlin-Heidelberg, 2011.

[5] H. Mühlenbein. The equation for response to selection and its use for prediction. *Evolutionary Computation*, 5(3):303–346, 1997.