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COMPROMISE-BASED APPROACH TO ROAD PROJECT SELECTION IN MADRID METROPOLITAN AREA

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Abstract This paper deals with an operational technique to specify the utility function in decision-making problems such as road project selection under non-strict uncertainty. We propose here a multi-criteria compromise programming (CP) proxy for utility. As a special case, the CP proxy turns into net present value. A case study on five highway projects aimed at improving transport opportunities in the Madrid metropolitan area is developed. After defining five suitable criteria or attributes for the model, these attributes (discounted value of savings in travelling costs, investment, rights of way, noise pollution, and urban capital gains) are specified in the random context for various states of nature. From this information, the compromise-utility proxy with market-based social weights is estimated, and expected utility maximized to rank the projects. Tables are incorporated to display the computational process.

Keywords: Decision making, transportation, Bayesian decision models, compromise programming, utility function, road project selection

1. Introduction

Currently, cost-benefit (CB) analysis, net present value (NPV), and internal rate of return (IRR) are standard approaches to rank long-range government transportation projects, which are often mutually exclusive. Both CB and NPV criteria require long-term projections of cash flows, which should be discounted back to the present by an interest rate, this rate being determined as the government's opportunity cost of capital. In CB, the analyst should estimate presumed benefits sometimes projected far into the future (which involves a high degree of uncertainty), sometimes intangible and indirect, which may involve the use of subjective criteria to determine and allocate qualitative values. These difficulties extend to present-value-based measures, such as NPV and IRR, even assuming certainty. Concerning IRR, this measure seems to be rather inappropriate to rank investment alternatives. Indeed, IRR relies on inadequate assumptions and violates essential principles: it does not satisfy the Fisher separation theorem and the value additivity principle, and the worst thing of all, it does not meet the requirement of discounting cash flows at the opportunity cost of capital (see [14] pp. 31-32). According to [14], NPV is "the only criterion that is necessarily consistent with maximizing shareholders' wealth," the property characterizing capital budgeting techniques. From the capital budgeting perspective, NPV shows great capacity as a sound comprehensive measure which consistently integrates many aspects of the project, providing that the market-determined opportunity cost of capital is chosen as the discount rate. However, the standard Bayesian approach to investor choice under uncertainty is not based on NPV, but on expected utility maximization ([14] Ch. 4; [15] Ch. 5). To apply the Bayesian model to governments' road projects, the question arises whether the utility function should be estimated subjectively or objectively. In the field of public decision making, the attempt to specify utility by eliciting weights from interactive dialogues between the analyst and a government official leads to nothing but to the use of subjective weights reflecting particular views. Conversely, market prices and other objective parameters reflect people's preference, namely, utility for society as a whole.

In this paper, a market-based specification of expected utility is proposed and applied to road projects in the Madrid metropolitan area. We assume that social utility cannot be subjectively specified by weights elicited from dialogues with government officials, but objectively specified from market prices involved in the projects insofar as these prices are related to social concerns, namely, social preferences and production costs. Apart from the weighting question, the road analyst faces other difficulties to specify utility (see Section 2 below). Most economists contend that linear or even additive utility forms should be rejected, because additivity entails independent utilities of the different attributes, which is a departure from realistic behavior ([23], p.15; [43]). In the multi-criteria approach here developed, the challenge of using non-linear utility in an operational way is overcome by resorting to the compromise-utility form which derives from compromise programming (CP), a methodology supported by a wide range of literature [45, 46]. Indeed, CP distance form is a meaningful general enough proxy for utility ([4], Ch. 6 and 8), easy to specify by preference weights, although it has not yet been applied to road projects. In summary, the proposed model is aimed at an easier practical application of Bayesian road project choice, a problem of great complexity to public and private decision-makers. Breakthrough techniques such as project finance have today changed critical features of investment decision making [5] but the road choice problem itself remains the same.

The paper is organized as follows. In Section 2, the Bayesian model is revised, emphasizing complexity and outlining the CP solution. Therefore, this part is oriented to explaining the principal methodological singularity of the paper. In Section 3, the attributes are defined and normalized; furthermore, the market- based weights are estimated. Modern techniques to estimate probabilities for Bayesian models are reviewed in Section 4 to facilitate their further use in the paper. Section 5 develops a real world case of five road projects recently considered by the Spanish authority to improve the highway network around Madrid. Some lines of future research are discussed at the end of the paper, along with a concluding remark.

2. Bayesian Model with Compromise Utility: Solving Difficulties of Application

Bayesian choice of the best solution among several alternatives is performed by maximising the expected utility of the outcomes, corresponding to the possible states of the world for each road or railway project. We first review the essence of Bayesian decisions applied to our environment. First, for simplicity, suppose the relatively trivial case of a single benefit attached to the alternatives, namely, the road projects. To select the best alternative in this simple case, we could resort to the following decision table:

Alternatives	States of the world					
(road projects)	χ_1	χ_2	$\cdots$	χ_j	$\cdots$	χ_n
a_1	z_{11}	z_{12}	$\cdots$	z_{1j}	$\cdots$	z_{1n}
a_2	z_{21}	z_{22}	$\cdots$	z_{2j}	$\cdots$	z_{2n}
$\cdots$	....	....	....	....	....	....
a_i	z_{i1}	z_{i2}	$\cdots$	z_{ij}	$\cdots$	z_{in}
$\cdots$	....	....	....	....	....	....
a_m	z_{m1}	z_{m2}	$\cdots$	z_{mj}	$\cdots$	z_{mn}

Each row corresponds to a course of action, namely, a road belonging to the opportunity set $(a_1, a_2, ..., a_i, ..., a_m)$. Let us define a single benefit such as the future traffic flow through the i^{th} road if this course of action a_i were selected. However, the decision-maker is not capable of achieving an exact forecasting about the traffic flow in the future. In fact, this variable depends on external factors, which are both unobservable and uncontrolled at the decision time. The number of external factors is often large. For example, the future traffic flow can be affected by factors such as future technology, rates of economic and demographic growth in the area, petrol prices, cultural preferences, and so on. Some of these factors behave as random variables with probability distributions allowing to estimate mean values, variances and trends. On the contrary, other factors are subject to uncertain significant changes, which cannot be described by a probability law. At best, generally speaking, likelihood parameters might be associated to the states of nature; that is, some uncertain variables could be treated in terms of the decision-maker's beliefs about their future levels. Indeed, changes in the variables are very difficult to predict in the long run, and often, also in the short run. In the decision table, even for the true states of nature being unknown, there is a reliable information on which states can happen. Moreover, a finite number of mutually exclusive states are commonly assumed.

The above decision table is a pay-off matrix describing the predicted traffic flow z_{ij} when the decision-maker chooses the i^{th} alternative while nature plays the j^{th} state. However, in Bayesian analysis, the decision- maker is not critically interested in the traffic flow level itself but in the consequences of this traffic flow. Such consequences are usually measured in terms of utility. Thus, the decision- maker's uni-argument utility function for the benefit considered (i.e., traffic flow) is embodied into the decision table rather than z_{ij}. In other words, instead of focusing on z_{ij}, the analyst focuses on the corresponding utility $U_{ij} = U(z_{ij})$. Function U is assumed to be strictly monotone increasing and moreover with decreasing marginal rate of growth (see e.g. [23], Ch. 2, p. 14).

If every likelihood p_j (for $j = 1, 2, ..., n$) has been estimated for the states of the world, the next step in the Bayesian approach consists of choosing the i^{th} alternative with maximum expected utility, that is:

$$\text{Max Expected Utility} = \max_i \sum_{j=1}^{n} p_j \, U_{ij}$$

In absence of probability or likelihood, Laplace's principle of insufficient reason, Wald's maximin-return rule, or other decision criteria for decisions under strict uncertainty (see [24], Ch. 13 and [15], Ch. 2) can be applied.

This scheme is called expected utility maximisation or EU(Z) optimisation theory [2, 32]. Indeed, Bayesian models rely on the mathematical properties derived from Von Neumann and Morgenstern elegant postulates [44], namely, the axioms of comparability, transitivity, strong independence, measurability and ranking.

The above statement raises some critical issues, which can be summarised in the following remarks:

(i) In real cases of project selection, the decision-maker does not face a utility function depending on a single argument (e.g., traffic flow) but faces a multi-attribute utility with plural benefits and costs [22]. Thus, we have

$$U(z_{ij1}, z_{ij2}, ..., z_{ijk}, ..., z_{ijq}).$$

Sometimes the number q of these attributes or criteria is high. Perhaps there will be a few attributes to estimate under certainty or at least, under risk with objective probability. For example, construction costs can be determined as an unrisky variable when there are fixed-price contracts for constructions. Regarding other attributes we can estimate likelihood although its estimation often becomes a difficult problem. Benefits such as the future traffic flow are subject to unexpected significant changes depending, e.g., on population preferences for living in new residential areas, new opportunities of road and railway lanes, location of new industrial activities and firms, etc. Even future habits and industrial ways of working (e.g., the rise of electronic workhome, or telework, in the happy new world) can drastically worsen the performance of any prediction based on probabilities. There is a long experience about failures in this type of prognosis. Therefore, the decision-maker can hardly consider the multi- attribute utility function for a large number of benefits and costs to incorporate into a very cumbersome decision table. Moreover, it is very well-known from psychological experiments that the human being is not able to distinguish among more than 5 variables once at a time, making almost impossible to assign multi-attribute and multi-variate functions [20].

To make clear the difficulties commented above, consider a decision-maker's utility function with two attributes. The specification of this utility requires to know the marginal rate of substitution (MRS) or trade-off relationship between both attributes at every point of the utility map [43]. Searching for these sets of MRS is not a trivial problem even by using sophisticated techniques of interactive dialogue (see [16, 30, 47]).

(ii) According to remark (i), the expected utility maximisation should be stated as follows:

$$\max_i \sum_{j=1}^{n} p_j \, U(z_{ij1}, z_{ij2}, ..., z_{ijk}, ..., z_{ijq}) \tag{1}$$

where $U(z_{ij1}, z_{ij2}, ..., z_{ijk}, ..., z_{ijq})$ is the multi-attribute utility function. This seems to be an intricate statement for most users and practitioners in road projects selection.

(iii) Some real problems do not have only one point of decision. They are typically dynamic, one problem leading to another, and that to a further, and so on. Moreover, these problems are interrelated, with the current possible actions and states being dependent on actions chosen in earlier problems. These are the often called *multi-stage* decision problems. Note that here we will have many kinds of uncertainties, and the probability to be assigned should be of a joint (multi-variate) form. Recently, new ways of representing decision analysis problems have been developed. The most important among them are the graphical models called decision trees and influence diagrams (see, e.g., [15, 19, 37]). All of them are mathematically equivalent to decision tables but are easier and intuitive to use. The joint probability distribution can be assigned through conditional and marginal probabilities; and we can even make them dependent on the alternatives, leading to more difficulties in the modelisation process and to a chain of maximizations (at each decision node) and expectations (at each chance node). See [6, 9], for a thorough analysis of the ongoing state of the art providing some solutions to many problems, as in [7], where a simulation-based approach to Decision Analysis is proposed.

(iv) To escape from this labyrinthine maximisation, the road analyst can assume additive independence, namely, a utility function that is the sum of the q uni- attribute utility functions U_k corresponding to the q attributes. Thus, the expected utility maximisation turns into:

$$\max_i \sum_{j=1}^{n} \sum_{k=1}^{q} p_j \, U_k(z_{ijk})$$

where p_j could also be dependent on i and/or k.

This simplification allowing to specify each uni-attribute utility separately from the other utilities, is not acceptable at least from a theoretical perspective. Economists contend that the additivity assumption is extremely unrealistic since the attributes in the bundle are interdependent. Moreover, even under the additivity assumption, the burden of searching remains very tedious (see, e.g., [15]). Most users are unfamiliar with the specification of utility functions, even with the specification of uni-attribute schemes. The effort required for this task generally is beyond the patterns and possibilities of authorities and firms which have road selection analysis on their responsibility.

(v) There is a further difficulty. Theoretically, in selecting a road project, the analyst does not focus on the preferences of a particular individual. In contrast, the utility function of a social group should be specified and optimised. According to Arrow's impossibility theorem ([2], or [15]) there is no constitution which permits to define the group preferences consistently by starting from the preferences of the group members. This new challenge can hardly be eluded by relaxing the axiom of no dictatorship (see, e.g., [15]) in the sense that the social group transfers the road decision responsibility to the government, and then the government itself transfers this responsibility to one official who makes a rational decision, see Section 3 below.

As noted in Section 1, the proxy for utility we use in this paper is CP, a multi-criteria decision technique to rank the activities from plausible assumptions. Implementation of this technique requires the following steps. Firstly, measurement and normalisation of the attributes, will be explained in Section 3 below.

Secondly, definition and specification of the ideal point should be performed. The ideal is a reference vector of normalised attributes with the coordinates $y_k^* = 1$ (for all k). Therefore, the best value of each attribute would be attained at the ideal. Generally, this reference point is infeasible as rarely an alternative reaches the best for all the attributes. From psychological studies [13], many decision-makers are willing to choose an alternative as close as possible to the ideal. In other words, the ideal point is a utopian aspiration so that CP model attempts to select the nearest alternative to this point.

Thirdly, the distance of this alternative from the ideal is computed although not necessarily by the Euclidean standard equation yielding the geometric distance between two points. In fact, the CP distance form is not commonly based on the quadratic metric but on other metrics. The most frequently used metrics for CP purposes, usually denoted by the symbol h, are either $h = 1$ or $h \to \infty$. The choice of a particular metric depends on the decision-maker's risk aversion ([4] p. 131). The more risk aversion the higher metric to use. Finally, the alternative of minimum distance is selected as the CP optimum solution, corresponding to the utility optimum.

Hence, we use the following decision function depending on the CP distance form:

$$U_{ij} = \frac{U^* - [\sum_{k=1}^{q} w_k^h (1 - y_{ijk})^h]^{1/h}}{U^* - U_*}, \tag{2}$$

for every i^{th} road and every j^{th} state of the world. In equation (2), the CP distance form is the term in square brackets. The symbols have the following meaning:

U_{ij} = CP ranking index for the i^{th} road, this index depending on the j^{th} state of the world;
y_{ijk} = normalised value of the k^{th} attribute for the i^{th} road and the j^{th} state;
w_k = weight assigned to the k^{th} attribute. Computation of these weights on an objective market price basis is explained in Section 3 below;
U^* = positive parameter sufficiently large to make (2) greater than 0. We take the greatest value of the distance form between square brackets in equation (2);
U_* = positive parameter lower than U^*. We take the smallest value of the distance form between square brackets in equation (2), thus getting $U_{ij} \in (0, 1)$;
h = CP metric.

Proxy (2) is justified as companies and organisations are willing to accept 'satisficing' solutions of 'bounded rationality' (in Simonian terminology) as analysed in classical research [39, 40]. 'Satisficing' is a neologism widely accepted in the multi-criteria literature. Some decision models such as CP, Goal Programming (GP) and Reference Point Method (RPM) have an underlying satisficing philosophy which is intuitively more appealing to practitioners [34] as well as easier to apply in real world environments. Equation (2) can be reduced to a utility form ([4], Ch 6). This linkage between CP and utility is significant enough to allow the use of equation (2) in Bayesian models. Whatever the utility function be, the bi-criteria utility optimum can be approximated by CP solutions since this optimum is proven to lie within the compromise set on which all CP solutions also lie ([4], Ch. 6). Although this property is not strictly extended to any number of attributes [10, 29] it considerably enhances the linkage between CP and utility.

The principal advantage of the CP form is easy specification of its parameters. As indicated, parameter h tends to ∞ for decision-makers with high risk aversion such as road companies and other big investors. Therefore, this parameter does not require specification. However, a sensitivity analysis concerning the impact of different metrics will be performed in dealing with the road ranking (see Sections 5.8 and 6).

In Figure 1, the utility setting is drawn. For the sake of simplicity, suppose there are only two attributes and also suppose $h = 2$, namely, the Euclidean metric. Corner points A, B, and C represent frontier feasible alternatives defining the convex set of road projects. If the attributes are normalized, then corner points A and C have coordinates $(1, 0)$ and $(0, 1)$ respectively. Therefore, road project A is characterized as exhibiting the best of the first attribute and the worst of the second one. Analogously, road project C exhibits the best of the second attribute but the worst of the first one. From these extreme alternatives we determine the ideal point $I(1, 1)$, an infeasible referential road project, which would contain the best of both attributes. If the utility index U is kept at a constant level U_0 in equation (2), we obtain an iso-utility curve, also referred to as indifference curve. For $h = 2$, the curve becomes an elliptic arc, the center of the ellipse being the ideal point. Two of these curves labeled $U^{(1)}$ and $U^{(2)}$ are portrayed in the figure. Notice that curve $U^{(2)}$ is "tangent" to the opportunity set ABC at point B. Hence, road project B is the utility optimum. If metric $h = 1$ were used instead of metric $h = 2$, then utility (2) would turn into the linear form, which often leads to unbalanced solutions such as A or C. This seems to be an additional reason to discard linear utility, as balanced solutions are commonly preferred.

To sum up, the key feature in the proposed Bayesian model is the use of compromise form (2) with a high-level metric, leading to balanced alternatives, which are suitable to risk averters. Thus, the cumbersome process of specification is considerably simplified and the main requirements of utility theory under uncertainty preserved.

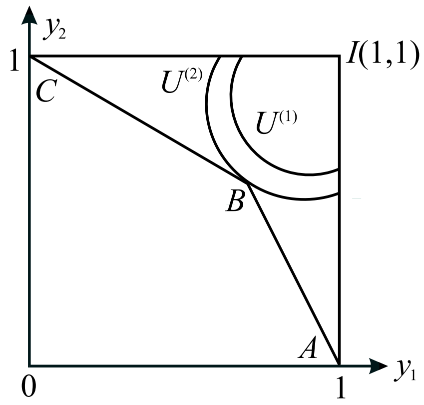

Figure 1: CP setting with Euclidean metric $h = 2$

3. Estimating Attributes and Market-Based Weights

3.1. Attributes

As a previous step to specify utility (2), each attribute should be measured in normalised units and each weight estimated in terms of social preferences. Let us start with measurement.

We will focus on a list of five attributes which will be used in our case study. They are labeled: discounted value of savings in travelling costs ($DVSTC$), investment, rights of way, noise pollution, and urban capital gains.

(i) $DVSTC$. This "more is better" attribute is defined in monetary units per kilometre as follows:

$$z_1 = \frac{GPV}{S} \tag{3}$$

where

GPV = gross present value of the road, namely, net present value plus investment costs plus rights of way costs;

S = straight distance from the beginning point of the road to the ending point.

Equation (3) is characterised saying that GPV means discounted benefits resulting from the road, namely discounted savings in travelling costs which are expected to be obtained by the users as a consequence of the new road. These savings consist of fuel, depreciation of vehicles, injuries from accidents and time wasted due to traffic congestion, road length, and worse opportunities. To estimate GPV, the analyst makes a comparison of opportunities before road and after road, taking into account intensity of traffic through the road in vehicles per day, savings in monetary units per vehicle, time horizon of the investment in years, and discount rate.

(ii) *Investment*, a "more is worse" attribute, that is defined in monetary units per kilometre as follows:

$$z_2 = \frac{IC}{S} \tag{4}$$

where IC is investment costs, which are borne by the sponsor. If IC were distributed over time, then, they would be computed in terms of discounted costs.

(iii) *Rights of way*, a "more is worse" attribute, that is defined in monetary units per kilometre as follows:

$$z_3 = \frac{RWC}{S} \tag{5}$$

where RWC is right of way costs, namely, payments for rights of way, which are borne by the sponsor. If RWC were distributed over time, then, they would be computed in terms of discounted costs.

(iv) *Noise pollution*, a "more is worse" attribute, that is defined in monetary units per kilometre as follows:

$$z_4 = \frac{SPC}{S} \tag{6}$$

where SPC means soundproofing costs, namely, aggregate costs of private investments in building soundproofing for real estates affected by noise pollution from the new road. Indeed, real estate owners may or may not invest in protection against noise pollution. If they are willing to invest, then, they will bear costs. If they are not, they will bear a decrease in the market value of their buildings, the quality of which is deteriorated by noise pollution. We then assume that decrease in market value can be measured as equivalent to soundproofing costs.

Hence, the following steps are required to estimate the z_4 attribute.

Start with the standard noise pollution equation (see [35]):

$$L_{10} = 80.6\text{dBA} + 9\,Log_{10}(I/20000) + 16\,Log_{10}(V/80) + 0.12\,(H-20) - 2.4\,(D-10)^{0.4} \tag{7}$$

where

L_{10} = noise level in decibels (dBA) exceeded 10% of the time,

I = intensity of traffic through the road, in vehicles per day,

V = mean speed of traffic (kilometres per hour),

H = ratio of heavy vehicles to total vehicles,

D = distance from near-side curb to the block of houses considered (metres).

Notice that equation (7) yields the noise level in each city block. From equation (7) we derive the minimum distance D so that noise pollution can be comfortably tolerated by residents living in the area beyond such distance. For this purpose, the level $L_{10} = 60$ dBA conforms to Spanish environmental legislation.

On the map of the service area we determine the urban surface Q extended D kilometres away from each curb of the road.

Aggregate soundproofing costs are computed by the equation:

$$SPC = cQ \tag{8}$$

where c means soundproofing costs per square metre of building. Therefore, as Q is the total urban surface affected by noise pollution, we obtain

$$z_4 = \frac{cQ}{S} \tag{9}$$

as a measure of the attribute under consideration.

(v) *Urban capital gains*, which generally behaves as a "more is better" attribute, defined in monetary units per kilometre by the following equation:

$$z_5 = \frac{UCG}{S} \tag{10}$$

where UCG means capital gains in real estates as a consequence of the road, namely, changes in the aggregate market value of real estates owing to construction of the road. Changes in price can result either positive or negative as depending on different economic/environmental impacts. Improvement in transport opportunities pushes market price upwards while landscape intrusion may have an environmental impact pushing prices downwards. Favourable effects of the road generally exceed unfavourable effects. Then, change in price is positive. In most cases, rise in price significantly depends on scarcity of roads in the area. The more severe isolation is the larger expected increase in the average price of real estates. However, one can think that landscape intrusion is also relevant and cannot be neglected in the analysis. In spite of this view, isolation, and not the environmental variable, seems to be the decisive factor determining the z_5 level, except in particular areas such as national parks and natural spaces specially protected. Indeed, two cases can be considered: (a) high density of roads in the area, then, the marginal impact of landscape intrusion will be small; (b) low density of roads, then, improving in transport opportunities will be preferred to mitigation of the environmental impact. Hence, we assume that z_5 behaves as a "more is better" attribute, which may plausibly be estimated by the following equation:

$$z_5 = \frac{v_{max}B/100S}{1 + \left[(v_{max}/\bar{v}) - 1\right](d/\bar{d})} \tag{11}$$

where

$B =$ market value of urban surface in the service area, before road (specified in monetary units);

$v_{max} =$ percent average increase in price of real estates in areas where density of roads is very high;

$\bar{v} =$ percent average increase in price of real estates in areas where density of roads reaches standard (mean) values;

$d =$ density of roads through and from the service area in the case of the road project under consideration (in kilometre per square kilometre);

$\bar{d}$ = density of roads as a standard (mean) value of densities in different areas of the region (in kilometre per square kilometre).

Equation (11) seems to be meaningful if applied to road investments within the same geographical context. Indeed, notice that $d = \bar{d}$ leads to $z_5 = \bar{v}B/(100S)$, namely, the standard mean increase in market price. Conversely, for zero-density we find the maximum increase in price. Moreover, the lower density the higher the capital gains, other things being equal.

3.2. Normalisation

Once every attribute has been defined and measured, its random consequence should be normalised. Certain random consequence z_{ijk} corresponds to each k^{th} attribute when the j^{th} state of the world occurs and the i^{th} alternative is taken by the decision-maker. Firstly, suppose that the attribute behaves as a "more is better" variable. In this case, the normalisation of the random variable is accomplished by the ratio:

$$y_{ijk} = \frac{z_{ijk} - z_{k*}}{z_k^* - z_{k*}} \tag{12}$$

where

y_{ijk} = k^{th}-normalised index for the j^{th} state and the i^{th} alternative,
z_{ijk} = k^{th}-non-normalised index for the j^{th} state and the i^{th} alternative,
z_{k*} = the worst (minimum) z_{ijk} for the k^{th} attribute,
z_k^* = the best (maximum) z_{ijk} for the k^{th} attribute.

Secondly, suppose that the attribute behaves as a "more is worse" variable. In this case, the normalisation of the random variable is achieved by the ratio:

$$y_{ijk} = 1 - \frac{z_{ijk} - z_k^*}{z_{k*} - z_k^*} = \frac{z_{k*} - z_{ijk}}{z_{k*} - z_k^*} \tag{13}$$

where

y_{ijk} = k^{th}-normalised index for the j^{th} state and the i^{th} alternative,
z_{ijk} = k^{th}-non-normalised index for the j^{th} state and the i^{th} alternative,
z_{k*} = the worst (maximum) z_{ijk} for the k^{th} attribute,
z_k^* = the best (minimum) z_{ijk} for the k^{th} attribute.

Through (13), the "more is worse" attribute z_{ijk} converts into the normalised variable y_{ijk}, which has a "more is better" significance. All normalised indexes are ranged over (0,1). Therefore, the best normalised value is $y_k^* = 1$ while the worst normalised value is $y_{k*} = 0$.

3.3. Market-based weighting

In examining the other side of the question, namely, the choice of weights, we discard any fashion of eliciting weights from preferences stated either by a single official or even by an official board. For the purpose of selecting road projects and other infrastructure projects, we realise that official preferences seem to be a rather inadequate measure for social utility. No general experience can be invoked guaranteeing that judges and values proposed by particular individuals be reliable estimates of judges and values accepted by the society as a whole. However, in competitive markets we find that prices are determined by interaction of supply and demand, which are closely related to production possibility frontiers and consumer's preferences. This fact leads to a plausible assumption that weights objectively based on market prices can reflect social utility more accurately than subjective weights, providing that the condition of competitive markets holds.

From the z_k market values, some market-based estimates of social weights can alternatively be proposed. They are:

(a) *Average-price weighting.* Here, the w_k weight is given by the following equation:

$$w_k = \frac{1}{m} \sum_{i,j} z_{ijk} p_j \tag{14}$$

where p_j means probability of the j-state of nature, so that the sum of probabilities for all j is equal to 1.

(b) *Range-based weighting.* Here, the w_k weight is estimated by the z_k range, namely,

$$w_k = z_{kmax} - z_{kmin} = |z_k^* - z_{k*}| \tag{15}$$

where z_{kmax} and z_{kmin} are the greatest and smallest values of the k^{th} attribute respectively. To justify range-based weighting, consider first the "more is better" case. From equations (12) and (15), we have:

$$w_k(1 - y_{ijk}) = z_k^* - z_{ijk}$$

namely, we just obtain the deviation of the z_{ijk} "more is better" attribute from its ideal value, this deviation being specified in monetary units. This result is meaningful from the social utility perspective. In fact, the dimensionless normalized deviation $(1 - y_{ijk})$ has just been reconverted into a disutility (deviation) specified in terms of market prices. Analogously, for the "more is worse" attributes. According to equation (2), all these partial disutilities are then articulated into the following expected utility:

$$E(U_i) = \frac{U^* - \sum_{j_1=1}^{n_1} \cdots \sum_{j_k=1}^{n_k} \cdots \sum_{j_q=1}^{n_q} p_{j_1} \cdot \ldots \cdot p_{j_k} \cdot \ldots \cdot p_{j_q} \left[\sum_{k=1}^{q} (z_k^* - z_{ij_k k})^h \right]^{\frac{1}{h}}}{U^* - U_*} \tag{16}$$

where for each $k \in \{1, ..., n_k\}$, $\sum_{j_k=1}^{n_k} p_{j_k} = 1$.

To rank the road projects, expected utility (16) is maximised. Equation (16) is appealing, not only by reason of using market-based weights (15), but also for other reasons. First, if the k^{th} range is zero, then the resulting indetermination form 0/0 is solved, turning into zero. This is also meaningful because a zero range attribute cannot influence the road ranking. Second, equation (16) turns into NPV for $h = 1$, namely, for linear utility. In the case study (Section 5 below), range-based weighting, and therefore equation (16) will be used.

4. Probability Issue

In this section we review the main methods to elicit probabilities to measure the uncertainty that we feel. Subjective probabilities are a cornerstone of the Bayesian decision analysis philosophy. The theoretical basis for judgmental probabilities is well established. These must be consistent with one's beliefs and for that reason there is no correct answer in the assignment process. However, since there are common errors when people informally determine probabilities and they tend to use heuristics that can lead to biases in assessment [20], a careful and formal protocol that guides and improves probability assessments is required [38].

There are three basic methods for eliciting probabilities of *discrete* events [12]. The first is simply to assign them directly. Obviously, the decision-maker may not be able to do it in many cases, e.g., if he is assigning the probability of a complicated event like one for a joint distribution or an event in which there is no much confidence. On the contrary, the decision-maker is required to respond in an indirect mode in the second and third methods. The second method consists of asking about the bets that the decision-maker would be willing to place. The answers will be given on the value scale (being, therefore, a V-method). The idea is to find a specific amount to win or lose such that the decision-maker is indifferent about which side of the bet to take. This indifference leads to have the same expected value of the bet regardless of which option is taken, thus deriving the probability in question. The process to find the indifference point is guided: a bet favors one side or the other depending on the response to the previous bet, making it more or less attractive until the desired point is reached. As an example, suppose we want to find out the probability of event E. The bets are:

	if E occurs	if E does not occur
Bet 1	win \$$X$	lose \$$Y$
Bet 2	lose \$$X$	win \$$Y$

and the decision-maker must find values X, Y such that he is indifferent between both bets. It would give $P(\text{E}) = Y/(Y + X)$.

The third method is based on comparing two lottery-like games and the answers will be given on the probability scale (being, therefore, a P-method). We usually find this scheme: suppose prize A is better than prize B and we ask the decision-maker to compare the following lotteries:

	win A	win B
Lottery 1	E occurs	E does not occur
Lottery 2	p	$1 - p$

Lottery 2 is called the *reference lottery* and the decision-maker has to find a value of p such that he is indifferent between the two lotteries. At that moment, p is the probability $P(\text{E})$ required. The process is carried out by changing gradually the value of p. For example, if starting with some p_1 the reference lottery is preferred, we should choose p_2 less than p_1 for the following round because the decision-maker considers that the chance of winning in lottery 2 is higher.

If there is no way to reach the indifference point, the decision-maker may assign a probability interval reflecting imprecision in his judgements. It leads to sensitivity analysis studies [33].

It is very useful to show the variation of the reference lottery at each question round. A typical and accepted device is a wheel of fortune although any other mechanism of randomisation could be used. In fact, many software packages include a wheel of fortune to aid in the assignment process. At `http://www.sis.pitt.edu/~dsl/da-software.html` most of the existing packages for probability elicitation can be found.

If we want to assign *continuous* probability distributions we can still use the previous ideas. For variable X, we can (1) fix various values x_i of X and assess the cumulative probabilities $P(X \leq x_i) = p_i$ by using the previous methods; or (2) fix various cumulative probabilities p_i and find the corresponding values x_i of the variable (i.e., the fractiles of the distribution). We can find these values by using lotteries as above replacing E with the event $X \leq x_i$. This option usually starts by assigning extreme values (5^{th} and 95^{th} percentiles), then the first and third quartiles and finally the median. For both methods (1) and (2), we can fit a smooth curve to the points obtained as an approximation to the cumulative distribution function.

Since these techniques do not work well for small probabilities, rare events probability encoding requires other methods. [42] is a standard reference on probability elicitation. Also, if we cannot assign some probabilities of *complex* events by using any of these methods, we can resort to decompose the event into other easier events [12], to use noisy OR-gates models to assign large conditional probability distributions with certain causal nature [31], and so on.

Once we have the continuous distribution, we have several possibilities to handle it. If we want to calculate an expectation, we can fit a standard distribution whose mean and probabilities are known. Another possibility is to use numerical integration or simulation techniques to approximate the expectation. We can also *discretize* the continuous distribution by finding representative points in the distribution and assigning them specific probability values. There are a number of different techniques: based on Gaussian quadrature [26], bracket-medians or bracket-means methods [12], centroid method [25], extended Pearson-Tukey method [21], moment-matching method [41].

If there are multiple decision-makers, we will use aggregation techniques. All the stages are formally stated in a standard protocol called SRI (Stanford Research Institute) [25]. It determines the judgements in a more exact manner than other methods commonly used for expert systems. All these techniques can be combined with historical data about the event in question, in case of having access to them.

Finally, the consistency among the interrelated probabilities must be checked since they should obey the probability laws. It is specially important in complex problems with many random variables. Also, some experiments can be carried out to determine whether a person is good at probability elicitation and whether is better after training, determining the *calibration* of the assigned probabilities with the real world, as in [1].

5. Case Study

In the metropolitan area of Madrid, the road and rail networks both converge on the town from all corners of Spain. A freeway system encircles the capital in an approximately pentagonal shape, coming to a point near Andalucía expressway. In 1994, the Spanish authorities were interested in some alternative projects for a new road connecting the M40 motorway around Madrid and the NV road near Navalcarnero. The latter town is located in the southern area of Madrid. The NV road is part of the European E-5, Madrid-Lisbon. The Spanish authorities had serious concerns regarding the traffic in the area, this traffic becoming quite congested in rush hours. Therefore, they were willing to construct another radial road to spread this traffic out. As a result of an efficiency analysis previously performed, the following five alternatives were considered for further selection, which was made by resorting to the NPV capital budgeting technique:

- Alternative R52 ($i = 1$). Its total length is 31.4 kilometres. It crosses near some crowded towns in Madrid southern area such as Alcorcón, Leganés, Móstoles and Fuenlabrada.
- Alternative R52a ($i = 2$). Its total length is 30.4 kilometres. It crosses a similar density area in the North side of R52 road.
- Alternative R52b ($i = 3$). Its first section along 23.5 kilometres coincides with R52 project. After this first stretch, R52b road crosses an area between R52 and R52a projects.
- Alternative R53 ($i = 4$). Its length is 26.7 kilometres. Starting from M40 road, it goes to the West side, crossing near the villages of Alcorcón and Villaviciosa de Odón. Also, it crosses Guadarrama River, and finally it arrives in Pozuelo town through stretches of heavily populated countryside.
- Alternative R53a ($i = 5$). Its length is 24.7 kilometres. Instead of arriving in Pozuelo town, it is located near NV old road.

As indicated in Section 3, we use the following attributes as suitable criteria to rank the road alternatives: $DVSTC$ ($k = 1$), investments ($k = 2$), rights of way ($k = 3$), noise pollution ($k = 4$), and urban capital gains ($k = 5$). Every attribute is considered as a random variable with exception of rights of way and urban capital gains, which can be determined for sure in the light of the available information. Regarding the random attributes, we assume that $DVSTC$ and noise pollution are strongly correlated, and therefore, an equal set of probabilities (in the sense of likelihood) is estimated for them. This assumption is justified since noise pollution and travel cost narrowly depend on intensity of traffic. Hence, the same states of nature and their respective probabilities are associated with both attributes, which are estimated by using standard approaches in road planning (see Subsection 5.7 below). Investment is a random variable depending on critical events that can occur throughout the construction period. In fact, the contracts commonly used in the Spanish road sector are categorised as "unit price contract". This type of agreement provides for bidders to insert their prices in a bid schedule listing the various items of work and their estimated quantities. A nominal change in the quantity of a unit price item is automatically taken care of, since the contractor gets paid for the actual number of units he constructs. Therefore, the sponsor/owner takes the risk of events affecting the random quantity of work for most items.

Process of computation is organised as follows.

5.1. Intensity of traffic

This is not an attribute, but a previous variable to compute some attributes in this section. From a survey performed by the Spanish road authority as a decision-making draft, the following information is obtained. Statistical sources for this information are simulation studies on regional traffic in the Metropolitan Road network of Madrid. To estimate the predicted traffic on the five road alternatives, the Spanish analysts made predictions of future population growth and economic activity, along with conjectures of their consequences on the land use and on travel requirements. These predictions were performed taking into account the various potential and existing transport alternatives in the metropolitan area, such as underground and surface railways (for example the Metrosur project). These railway projects will be extensions from the underground network, which serves the capital with twelve lines that spread out on the town.

Some of the radial motorways are now crowded. A road plan exists to build considerable stretches of new high-speed roads alongside these multilane intercity highways. To estimate the predicted growth of traffic flow through the roads, a common assumption is used such as traffic between two towns is roughly proportional to their expected populations and inversely proportional to the distance between them. The Spanish road authority gave a technical survey on the predictions of traffic flow over the period 1996-2020 (see [27]). For the five road alternatives focused on our case study, the results are shown in Table 1, in which three states of nature are considered. In Figure 2 these results are graphed.

Table 1: Predicted growth of traffic flow over 1995-2020 (1995=100)

Year	1996	97	98	99	2000	1	2	3	4	5	6	7
	108	113	118	123	127	132	136	138.5	142	145	147.5	150
	104	107.5	112	115	118.5	122	125	127	130	132.5	135	137.5
	99	101.5	104	107	110	113	115	117	120	122	124	125.5

8	9	10	11	12	13	14	15	16	17	18	19	20
152	154	156	158	160	162	163.5	165	166.5	168	169	170	171
139	141.5	143	144.5	146.5	148	149	150	151	152	153	154	155
127	129	130	131.5	132.5	133.5	134.5	135.5	136.3	137	137.5	137.8	138

Rows are states of nature $j = 1, 2, 3$

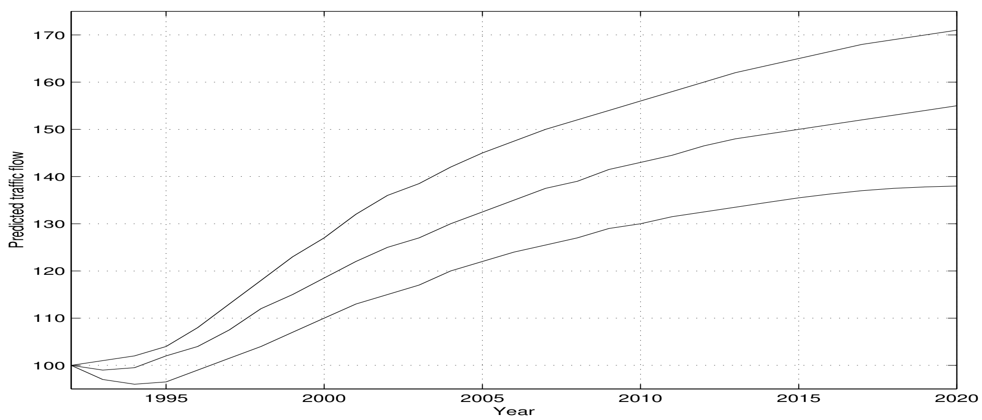

Figure 2: Predicted growth of traffic flow over 1995-2020 (1995=100)

From a second survey [28], a reliable prediction information on traffic flow through the five road alternatives is obtained (see Table 2). This prediction refers to 1995 (the selected alternative is going to be constructed after the year 2000).

Table 2: Predicted traffic flow through five road alternatives (1995)

Alternative i	1	2	3	4	5
Traffic flow	24700	25100	25000	20000	20000

Vehicles per day

From Tables 1 and 2, the discounted values of traffic flow referred to the year 2000 are computed by using 3 percent as a suitable rate of discount. Each discounted value is divided by 25, namely, the number of years from 1996 to 2020, see Table 3.

Table 3: Discounted values of traffic flow (year 2000)

$k=1$	States of nature	Road alternatives $i=1$	$i=2$	$i=3$	$i=4$	$i=5$
	$j=1$	27595.717	28042.611	27930.887	22344.710	22244.710
	$j=2$	25405.957	25817.389	25714.531	20571.625	20571.625
	$j=3$	23274.786	23651.706	23557.476	18845.981	18845.981

Vehicles per day

5.2. *DVSTC* (z_{ij1})

This attribute is estimated as explained in Subsection 3.1, paragraph (i). To compute GPV, in millions of deflacted Spanish pesetas, the Spanish analysts used empirical information published elsewhere [27]. This information deals with fuel, depreciation of vehicles, accidents, and opportunity cost (or "time is money" cost) for travellers. Time horizon and discount rate have been taken equal to 25 years and 5% respectively. The results appear in Table 4.

Table 4: *DVSTC*

$k=1$	States of nature	Road alternatives $i=1$	$i=2$	$i=3$	$i=4$	$i=5$
z_1	$j=1$	1803.042	1827.064	1786.875	1160.102	1060.746
	$j=2$	1615.111	1636.63	1600.63	1039.185	950.185
	$j=3$	1432.209	1451.291	1419.367	921.503	842.582

Source: Official survey [28]. More is better

Therefore, the z_1 results are specified in millions of deflacted Spanish pesetas per kilometre, the S straight distance (27 kilometres) being the same for the 5 road projects.

5.3. Investment (z_{ij2})

Approach in Subsection 3.1, paragraph (ii) is here applied. As a first step, we estimate predicted levels of investment costs for 5 states of nature. Type of construction agreement is here a relevant factor to take into account. Possible types of agreements are: Lump sum contract, Unit price contract, Cost reimbursable contract, Design build contract, etc. However, unit price contract is the standard arrangement for roads in Spain. Completion Risk is partially allocated to the engineering and construction contractors, although the sponsor/owner also takes a considerable part of such risk. Nevertheless Completion Risk can be discarded in the context of our case study. Cost Overrun Risk is assigned to the sponsor/owner insofar as this risk derives from the random quantity of work. Payments to the constructor are distributed over time, and consequently, investment cost is computed as a present value of these payments.

In Table 5 the states of nature as for investment cost overrun risk are considered with their corresponding levels in millions of deflacted Spanish pesetas. Therefore, the z_2 results are specified in millions of deflacted Spanish pesetas per kilometre.

Table 5: Investment

$k = 2$	States of nature	Road alternatives $i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
z_2	$j = 1$	914.3	904.4	906.2	844.7	751.9
	$j = 2$	965.1	954.7	956.5	891.6	793.6
	$j = 3$	1015.9	1004.9	1006.9	938.5	835.4
	$j = 4$	1066.6	1055.2	1057.2	985.4	877.2
	$j = 5$	1117.4	1105.4	1107.6	1032.4	918.9

More is worse

5.4. Rights of way (z_{ij3})

Approach in Subsection 3.1, paragraph (iii), is applied. As noted above, this is a risk-free attribute, and therefore, the sole state $j = 1$ is considered. "Right of way" costs substantially include compensations to real estate owners whose urban land is purchased by the sponsor either from voluntary transfers or by means of expropriation. Since payments to land owners are distributed over time, right of way cost RWC is computed as a present value of these payments. Estimation is shown in Table 6, from a survey given by the Spanish road authority [28]. Therefore, the z_3 results are specified in millions of deflacted Spanish pesetas per kilometre.

Table 6: Rights of way

$k = 3$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
z_3	62.74	65.41	62.89	105	13.67

More is worse

5.5. Noise pollution (z_{ij4})

In the Madrid region, most people live in large towns around the capital where the pace of life is fast, vehicles are everywhere, and environmental noise is a major problem. As shown in Subsection 3.1, paragraph (iv), the intensity of sound is measured in logarithmic units, termed decibels (dBA). Many houses in the area do not meet minimum standards of construction to provide sufficient shielding from external sounds. A change from a level of 50 decibels to one of 60 decibels involves a 100-fold increase in the sound level. To find the D distance for each road alternative, this D is taken out from equation (7). From Spanish road surveys, the variables other than D in equation (7) are assigned the following numerical values: $L_{10} = 60$ decibels, $I =$ intensity of traffic in vehicles per day (see Table 3), $V = 110$ kilometres per hour, and $H = 10$ per cent. After this step, the Q urban surface affected by noise pollution is computed in function of each D value for every road alternative. From architecture projects in the region, we estimate soundproofing costs as $c = 3750$ Spanish pesetas per square metre of building. In Table 7, the above information is recorded, leading to the z_4 values given by equation (9), which are specified in millions of deflacted Spanish pesetas per kilometre.

5.6. Urban capital gains (z_{ij5})

Computation of z_5 is carried out from equation (11) with the following numerical data: $v_{max} = 17$ percent, $\bar{v} = 13$ percent, $\bar{d} = 0.13$ kilometres per square kilometre, these values coming from road projects and road maps in the region. Remaining necessary information

Table 7: Noise pollution

$k=4$	States of nature	Road alternatives $i=1$	$i=2$	$i=3$	$i=4$	$i=5$
	$j=1$	142.254	143.072	143.192	37.58	50.12
	$j=2$	140.765	141.584	141.704	36.659	49.222
	$j=3$	139.185	140.004	140.125	35.679	48.27

More is worse

on B and d for each road project is shown in Table 8, leading to the z_5 values specified in millions of deflacted Spanish pesetas per kilometre.

Table 8: Urban capital gains

$k=5$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$
B	177187.5	181723.5	180778.5	112738.5	111132
d	0.184	0.18	0.183	0.146	0.15
z_5	777.56	802.29	794.34	527.79	516.02

More is better

5.7. Probabilities

To obtain the expected utilities, we have the information from the above tables. However, the probability assignment of the states of nature in Tables 4, 5 and 7 remains to be done. These states of nature correspond to the following uncertain variables: $DVSTC$, investment (or equivalently IC) and noise pollution. Remember that the same states of nature and their respective probabilities are associated with $DVSTC$ and noise pollution due to their strong correlation. The process to reach three levels for $DVSTC$ and five levels for IC together with their probabilistic quantification was carried out through formal elicitation interviews with the decision-maker. Since there were not enough historical data to determine probabilities, we proceeded by extracting and quantifying judgmental (subjective) probability distributions.

Following the standard SRI protocol, we first motivated the decision-maker about the importance and purpose of the encoding task in this problem. This motivation and the consequential conclusion were performed by one of the authors (JMA) as former official in the Spanish Road Office (Ministerio de Fomento, Dirección General de Carreteras) in cooperation with other officials of the Department. For this task, the above quoted working survey [28] was consulted. Moreover, since the decision-maker is going to have neither any personal reward nor commitment, we did not find any motivational biases influencing his probability elicitations. During the structuring stage, both $DVSTC$ and IC were defined as unambiguous variables passing the so called *clairvoyance test* (see e.g. [18]). As shown above, their measuring scales were millions of deflacted Spanish pesetas per kilometre for $DVSTC$ (and noise pollution), and millions of deflacted Spanish pesetas for IC.

The purpose of the conditioning stage is to find out what assumptions the decision-maker is making about the uncertain variables for the assessment and to bring into the decision-maker's mind the relevant knowledge about these variables. We took into account that all relevant information was considered, and not just that which immediately came to decision-maker's mind, thus combating cognitive biases like the *availability* bias [20].

Next, the SRI protocol to estimate subjective probabilities for one of the cases considered (IC) is presented. The analyst dealt with the remaining cases in a similar way. We began the encoding stage asking first for what he considered to be extreme values of IC (0.99 and 0.01 fractiles): "Think of a cost in millions of deflacted Spanish pesetas that is high enough so that it is not very likely to be exceeded, although it could happen. In quantitative terms, you can think of something like a one in a hundred chance". He answered 30000 millions of deflacted Spanish pesetas after some questions on the reference lottery to reach this indifference point. The process is summarised in Table 9.

Table 9: Dialog to determine a 0.99 fractile

		win A	win B	preferred
1^{st} round	Lottery 1	$>$ 2000 occurs	$<$ 2000 occurs	Lottery 1
	Lottery 2	0.01	0.99	
2^{nd} round	Lottery 1	$>$ 40000 occurs	$<$ 40000 occurs	Lottery 2
	Lottery 2	0.01	0.99	
3^{rd} round	Lottery 1	$>$ 20000 occurs	$<$ 20000 occurs	Lottery 1
	Lottery 2	0.01	0.99	
4^{rd} round	Lottery 1	$>$ 30000 occurs	$<$ 30000 occurs	Indifferent
	Lottery 2	0.01	0.99	

17000 millions of deflacted Spanish pesetas was the answer for the low extreme value, i.e., 0.01 fractile. Next we shifted to working on the center of the distribution. We took a set of values for IC and used the probability wheel to encode the corresponding probability levels. For example: "Let us look at the kind of numbers you tend to plan on and try to attach probabilities to them. What is the probability that 27000 millions of deflacted pesetas would be exceeded?" With the aid of the wheel, he answered that it is quite high, as three-quarters. Thus, we had the third quartile. Other questions and answers led to assign 21000 million pesetas as the first quartile. Next, as a check, we asked for values (rather than for probabilities) close to 21000 and 27000 to verify numbers.

We then asked for the median (0.5 fractile): "Do you think you are more likely to find an IC greater than 26000 millions of deflacted pesetas or less than 26000?" With the wheel, he pointed out less than 24000. Thus we knew that the median is lower than 26000. Finally, the decision-maker could not say more than "it would be somewhere between 24000 and 26000". We confirmed with him that previous numbers agreed with this and decided to start the verifying stage.

Some questions tried to refine more the median, e.g. "if you had to split the range into two equally likely pieces, where do you split?", with the answer: 25500 after some trials with lotteries. Other questions tested that the specified interquartile range had a probability of 0.5: we realised that the lower quartile should be 22000 rather than 21000 since the decision-maker assured that more than fifty percent of the time you were going to be inside the range and less than fifty percent of the time outside. We confirmed the consistency of both quartiles checking that it was about equally likely to be in the range above the upper quartile and below the lower quartile.

We plotted the cumulative probability distribution by fitting a curve to the points just obtained, see Figure 3(a). We used this function to determine an approximate probability density function to use in further consistency checks, see Figure 3(b). We presented the decision-maker a sequence of pairs of bets drawn from the distribution curve so that each pair should be equally attractive, and he gave indifference responses. We finally had confidence that the curve represented his judgements.

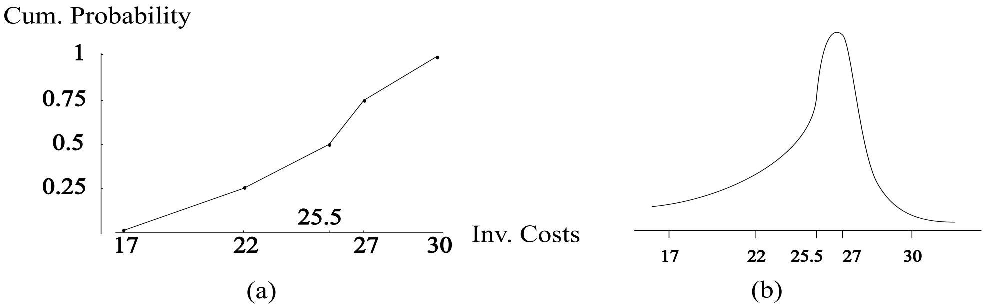

Figure 3: (a) Cumulative distribution function of investment cost; (b) Approximate density function. X axis units $\times 1000$

Similarly, the decision-maker elicited the other random variable $DVSTC$. The time (1.5 hour) and effort expended in probability encoding was worthwhile because of the importance of these variables and this decision problem. Often some responses showed the need for assignment revision and we had to return to previous phases.

In order to achieve a computational simplification when computing expected utilities, we decided to discretize these continuous probability variables. As noticed in many discretization methods, it is usually important not to distort the mean, variance and other moments of the original distribution. We used the centroid method [25] which minimises discretizing errors for problems having a smooth utility, like in our case. Hence, for IC we derived the discrete distribution shown in Table 10, where $j = 1$ was the interval "less than 17500 millions of deflacted Spanish pesetas"; $j = 2$ was "between 17500 and 21000"; $j = 3$, "between 21000 and 26000"; $j = 4$, "between 26000 and 28000" and $j = 5$, "between 28000 and 31000 millions of deflacted pesetas".

Table 10: Probabilities of the states of nature for IC

State of nature	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$
Probability	0.05	0.15	0.40	0.25	0.15

For $DVSTC$ (and noise pollution), three levels were sufficient and we chose the intervals with about 25% of the probability each assigned to "high" ($j = 3$) and "low" ($j = 1$) and the remaining probability (50%) to the "medium" region ($j = 2$). These levels come from the above quoted handbook on benefit-cost for Spanish roads [27].

5.8. Results

Finally, equation (16) is applied to estimate the expected utility for each road project. From Tables 4-8, we obtain the z_k^* and z_{k*} values, $\forall k$:

k	1	2	3	4	5
z_{k*}	842.582	1117.437	105.000	143.192	516.018
z_k^*	1827.064	751.867	13.667	35.679	802.287

As far as U^* and U_* are concerned, they depend on the metric. For example, for $h = 2$, these values are $U^* = 1038.86$, $U_* = 193.615$. They have been computed as explained in their definitions below equation (2).

Numerical values of data and results for different CP metrics appear in Table 11.

Table 11: Expected utilities of each road i, for different CP metrics h

	$E(U_1)$	$E(U_2)$	$E(U_3)$	$E(U_4)$	$E(U_5)$
$h \to \infty$	0.811	0.828	0.813	0.237	0.13
$h = 10$	0.809	0.826	0.81	0.237	0.13
$h = 5$	0.802	0.821	0.803	0.238	0.131
$h = 2$	0.771	0.793	0.771	0.207	0.131
$h = 1.5$	0.747	0.774	0.748	0.171	0.137
$h = 1$	0.712	0.756	0.72	0.159	0.226

When metric $h \to \infty$ is chosen, we find in this table that the best project is road R52a, as corresponding to the highest expected utility index $E(U_2)$, in column 2. The Spanish authority made its decision by implementing the NPV criterion from techniques like those explained in Sections 5.1-5.4, selecting thus the same road project.

A sensitivity analysis for metrics other than the infinite metric is shown in the other rows of the table. Especially those metrics greater than the Euclidean $h = 2$ may become good candidates, as metrics less than 2 involve quite low risk aversion. Note that the road ranking does not change with the metric chosen, if this metric is greater than (or equal to) 1.5. For metric $h = 1$, the ranking slightly changes as $E(U_5)$ is greater than $E(U_4)$, this relationship resulting inverse if any other metric is used. As noted in Sections 2 and 3.3, metric $h = 1$ leads to linear utility, which is rather inappropriate to achieve a reliable ranking from expected utility with risk aversion. Since NPV yields the same ranking as $h = 1$, the analyst should not rely on net present value without comparing the NPV solution to those derived from non-linear expected utility involving risk aversion.

Time simplicity of the algorithm used to compute expected utility (16) should be pointed out. Optimization of the root-form (16) by Mathcad7 (`http://www.mathsoft.com`, Mathsoft International), a symbolic calculus program with an algebraic-like language, is achieved in a few seconds. Conversely, the more time-consuming process has been to elicit probabilities and to search for z_k numerical values. Once with all this empirical information, a simple computational process on a 200-MHz Pentium PC came up with the final result. A slightly smaller burden is required with NPV.

6. Concluding Remarks

A Bayesian model should be used wherever possible for road project choice, as expected utility maximisation is the standard methodology to rank alternatives under uncertainty. However, people such as road and railway project decision- makers are not currently impressed with Bayesian utility functions, and therefore, any reassurance on applicability that we can provide would be helpful. The CP proxy for utility that is proposed in this paper seems to be advantageous to facilitate specification of the objective function from market-based weights as displayed through the case study in a real world scenario.

Future improvements/extensions are now suggested.

(i) *Probabilities.* The uncertainty assignment process could be relaxed by allowing the decision-maker to provide probability intervals reflecting imprecision in judgements. As to attributes such $DVSTC$ and investment costs, we might use elicited continuous probability distributions, with expected utility being approximated by Monte Carlo methods. (ii) *Alternative utility forms.* In place of compromise-utility (2), partially assessed utilities and fuzzy measures [17] could be viewed as suitable tools for the Bayesian model. Concepts like non-dominated alternatives can be useful [8]. (iii) *Future additional lanes.* Their construction in the future could be foreseen by changing the levels of attributes such as investment and rights of way, both to be measured as random variables. (iv) *Urban capital gains.* Approach (11) might be extended by undertaking a regression analysis on the UCG levels as observed over the past years from wide samples of road projects in the region. (v) *Attributes.* Their list could be enlarged. Moreover, rights of way and UCG might be treated as random variables. (vi) *Choice under strict uncertainty.* If facing decision tables where probabilities are unknown, neither expected utility nor capital budgeting techniques can be used. Then, we can resort to a theorem appeared elsewhere [3], which allows us to measure the uncertain attribute by a consistent system of weights from a moderately pessimistic perspective.

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