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Bayesian Methods in Reservoir Operations: The Zambezi River Case

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Abstract

This paper describes a general Bayesian methodology for reservoir operations. This methodology is applied to the operation of the two main reservoirs in the Zambezi river: Lake Kariba and Cahora Bassa.

KEYWORDS: Reservoir operations, Dynamic models, Multiattribute utility, Dynamic programming, Sensitivity analysis.

1 Reservoir operations

Many reservoirs have been built and are operating under different climatic and hydrological conditions, and with many different purposes, including municipal, industrial or agricultural water supply, hydropower generation, flood protection and control and recreation. In fact, many reservoirs serve several conflicting purposes, as in the case of having to secure water supply to a number of users and to provide flood protection downstream.

Multiple objectives are not the only difficulties associated with reservoir operations. The inflow process is typically uncertain and with varying dynamics. Water demand and priorities may also change from one period to another. Consequences of operational decisions might affect different groups of people in different ways. These and many other features of reservoir operation problems cause that, despite a long tradition of efforts aimed at developing efficient and secure methods for reservoir operation, no generally accepted methodology is available yet.

In this paper, we provide a general Bayesian methodology and describe its application to the main reservoirs in the Zambezi river.

1.1 Quantitative methods in reservoir operation

The literature dealing with various aspects of single and/or multiple reservoir management is rich and covers a broad range of methods. Since there is no single, general formulation for the problem, there is no universally applicable solution approach. We shall concentrate on reservoir operation problems, leaving aside related problems of reservoir design (site and size). Thus, our problem is: given an existing reservoir, find the operating policy or set of rules specifying how much water has to be released from the reservoir for different purposes and at various instants of interest.

The very first methods in the field developed in the 19th century were based on simple diagrams, Klemes (1981). Since then, both single and, much less frequently, multiple objective optimization methods have been proposed. Also, uncertainty has been included, both explicitly (via probabilistic models and techniques) and implicitly (via scenarios). We believe though that the literature dealing with reservoir operation problems typically identifies operation with a specific technique applied to solve a certain formulation of the problem, leading to the false conclusion that the reservoir operation task can be solved by direct application of the optimization method described in a paper. This 'technique focused' approach fails to present a comprehensive framework necessary to tackle reservoir operation problems without too many simplifying assumptions. The most common defficiency of various 'technique focused' approaches boils down to a lack of mechanisms for using incoming information about inflow and users' demands and preferences, thus limiting the use of up to date information concerning reservoir operation conditions.

1.2 Bayesian methods in reservoir operation

We aim at solving the problem within the Bayesian framework. That would require:

- defining the set of control alternatives, in our case release policies u;
- providing a forecasting model for inflows i to the reservoir;
- modelling the consequences $c(u, i)$, associated with policy u and inflow i;
- modelling the preferences of the decision maker by means of a utility function F, typically multiobjective and multiperiod;

Let h designate the predictive density of inflows and D the inflow history. Let subindex j refer to period j, s_{t+k+1} be the final storage, and k the planning horizon. Then, at time t, the reservoir management planning problem consists of finding the controls $(u_t, ..., u_{t+k})$, maximising the expected utility

$$\int F(c(u_t, i_t), ..., c(u_{t+k}, i_{t+k}), s_{t+k+1}) h(i_t, ..., i_{t+k} | D_t) di_t ... di_{t+k}$$

taking into account the dynamics of the reservoir system, and the constraints over controls and reservoir storages. The problem becomes unmanageable for a realistic planning horizon, say 36 months, since we have to solve a long term stochastic dynamic program; the evaluation of each control requires the solution of a high dimensional integral; and uncertainty about the inflow process rapidly propagates through time.

Instead, we adopt a strategy based on a concept of 'reference trajectory', which assumes having found a good 'reference' storage level for each period. Then, at each period we would like to maximise the expected value of a utility function F^* taking into account the consequences of interest and the deviation from that reference state, i.e.,

$$\int F^*(c(u_t, i_t), \delta(s_{t+1}, s^*_{t+1}))h(i_t|D_t)di_t,$$

where $\delta(s_{t+1}, s^*_{t+1})$ represents the deviation of the final state s_{t+1} from the reference state s^*_{t+1}. Intuitively, if the reference states are defined in such a way so as to account for the dynamic aspects of the problem, we would not loose too much with this approach.

This proposal is, in fact, based on a traditional and well established method of reservoir operation using the concept of *rule curves*, which represent optimal trajectories of the reservoir over a long time horizon, see Loucks and Sigvaldasson (1982).

2 The Zambezi river case study

The Zambezi is the largest African river flowing into the Indian Ocean. Its total catchment area covers 1300000 km^2, while its length from the source in the Central African Plateau to the outlet is in the order of 2500 km (Balek, 1977). The catchment is shared by eight riparian countries; for Mozambique, Zambia and Zimbabwe it constitutes the main water resource. The main use of the river is hydropower generation. The main reservoirs are Lake Kariba and Cahora Bassa.

Lake Kariba, on the border between Zambia and Zimbabwe, is, currently, the fourth largest man-made lake in the world. At the maximum retention level it covers an area of over 5600 km^2 and has an active storage exceeding 70 km^3. Hydropower plants installed at the northern (Zambian) and southern (Zimbabwean) banks of the dam plus a small hydropower scheme located on the Kafue River jointly supply more than 70 % of the electricity produced in these two countries. Since the completion of the generating facilities in 1977, the Lake Kariba system has supplied a monthly average of about 600 GWh/month, with little seasonal variation. Both countries operate the scheme jointly and share the electricity generated on a 50–50 basis.

Cahora Bassa is located in Mozambique. The maximum area is 2700 km^2. The maximum impounded water is 60 km^3. The energy produced is mainly sold to South Africa. However, until recently, the transmission lines have been a favourite terrorist target, limiting its operation.

Figure 1 here

Figure 1: The Zambezi river basin

2.1 The IIASA project

Upon request from the governments of Botswana, Zambia and Zimbabwe, the United Nations Environment Program (UNEP) assisted the Governments of the Zambezi basin countries developing the Zambezi action plan (ZACPLAN). In May 1987, an agreement on the *Action Plan for the Environmentally Sound Management of the Common Zambezi River System* was adopted and signed in Harare (Zimbabwe), by representatives of five basin countries (UNEP, 1987). ZACPLAN was then approved by the summit of the Southern African Development Coordination Conference and a special unit was established to implement the plan.

At the same time, UNEP recognised the need to improve government decision making processes, in selecting the best environmentally sound alternatives in planning and managing large international water systems. The International Institute for Applied Systems Analysis in Laxenburg (Austria) was then asked to contribute to the work and develop appropriate methods and tools that could be used to support decision making processes associated with the management of Zambezi resources (Salewicz and Loucks, 1988; Salewicz, Loucks and Macdonald, 1989).

IIASA studies in connection with the Zambezi case covered several issues (Pinay, 1988) but focused mainly on two topics:

- Development of an interactive river system simulation program, called IRIS, now in use in several countries (see Venema and Schiller, 1995).

- Development of models and methods to improve the operation of the main hydropower scheme in the Zambezi basin at Lake Kariba. Gandolfi and Salewicz (1990, 1991) conducted extensive studies and developed two sets of operating policies for the scheme, combining simulation and multiobjective optimization, which improved upon the current management.

These studies were the origin of the project described here.

3 Bayesian methods in Lake Kariba and Cahora Bassa operation

In Ríos Insua and Salewicz (1995) (RS from now on), we explored the potential of the Bayesian framework described above and improved significantly the performance of Lake Kariba. Here we demonstrate the applicability of our framework to multireservoir systems with the case of Lake Kariba and Cahora Bassa. As explained in Lamond et al (1995), stochastic optimization of multireservoir river basin models is bedeviled by the 'curse of dimensionality'. We suggest our methodology as a powerful alternative.

In the study, one of us (Salewicz), drawing on experience of the operation of the Lake Kariba over an extended period, plays the role of an expert providing beliefs and preferences, which are then thoroughly checked via sensitivity analysis. This is a common practice in public policy decision analysis (Keeney, 1992).

As notation is concerned, superindex c will refer to Cahora Bassa, whereas superindex k will refer to Kariba Lake. For example, i_t^c and i_t^k will designate the inflows (in mln.m^3/ month) to Cahora and Kariba at month t, respectively.

3.1 Operating policies

Let us start defining our operating policies. Current ones are defined by means of parametric curves associating release with the amount of water stored in the reservoir. The parameters of these curves were chosen to optimise some performance function. Instead, we prefer to formulate more flexible rules, as described for Cahora Bassa:

- At the beginning of the month, the operator announces the desired amounts of water u_{1t}^c and u_{2t}^c (in mln.m^3/month) to be released, respectively, for energy production and for free storage provision to catch expected floods.
- If there is not enough water to release u_{1t}^c, all available water is released for energy production. Otherwise, u_{1t}^c is released for energy production.
- If, after the release of u_{1t}^c, there is still water available, some water may be additionally released to control the reservoir storage level. If there is not enough water to release the volume u_{2t}^c announced, all available water is released. Otherwise, u_{2t}^c is released. In the event that, after the two releases, the remaining water would exceed the maximum storage $M^c = 60000$ mln.m^3, all excess water is spilled.

For Kariba Lake, there are corresponding releases u^k_{1t} and u^k_{2t}, defined in a similar vein, with maximum storage $M^k = 70980$ mln.m^3.

We shall find optimal controls $u^c_1, u^c_2, u^k_1, u^k_2$ associated to our model. Note that actual releases will differ from the announced ones, depending on the available water.

3.2 Consequences of operating policies

¿From the operational and managerial viewpoint, the consequences of an operating policy for Cahora Bassa at the end of every month are:

1. the amount of energy produced E^c_t in GWh/month,

2. the reservoir storage s^c_t in mln.m^3.

For Kariba the relevant consequences are:

1. the existence of energy deficit k_t, with a target of 750 GWh/month,

2. the volume of spilled water u^k_{2t},

3. the volume of water released $u^k_{1t} + u^k_{2t}$,

4. the reservoir storage s^k_t.

Note the differences for both reservoirs: first, energy at Cahora Bassa would be sold eventually to other countries, whereas energy at Kariba is consumed within the producing countries. The second consequence of interest for Kariba is due to an objective related with homogenity in operation through time, for reasons described in Gandolfi and Salewicz (1991). The third one relates to the need to take into account the release from Kariba in the inflow model to Cahora Bassa. Reservoir storages are required to keep track of the evolution of reservoirs through time.

An important feature in the problem is the dynamics of the basin. Here we describe Cahora Bassa dynamics. Interreservoir dynamics are described in Sections 3.3.2 and 3.3.3. Lake Kariba dynamics are described in RS.

Let u_t^c, e_t^c denote, respectively, the amounts of water released and evaporated during month t, with i_t^c and s_t^c defined as above. A continuity equation describes the relation between storage level, inflow, outflow and evaporation:

$$s_{t+1}^c = s_t^c + i_t^c - u_t^c - e_t^c.$$

Total outflow is

$$u_t^c = u_{1t}^c + u_{2t}^c. \tag{1}$$

For evaporation, we use a model

$$e_t^c = m_t \left(a \times \frac{s_t^c + s_{t+1}^c}{2} + b \right),$$

where m_t represents evaporation intensity during month t. Simple computations lead to a new version of the continuity equation:

$$s_{t+1}^c = d_{1t}^c s_t^c + d_{2t}^c (i_t^c - u_t^c) + d_{3t}^c, \tag{2}$$

where d_{1t}^c, d_{2t}^c, d_{3t}^c are appropriate periodic constants obtained from simple transformations and are in Table 1.

Month	d_1^c	d_2^c	d_3^c
Oct.	.994	.997	989.640
Nov.	.996	.998	990.375
Dec.	1.000	1.000	992.395
Jan.	1.000	1.000	992.476
Feb.	1.000	1.000	992.807
Mar.	.998	.999	991.691
Apr.	.996	.998	990.772
May	.996	.998	990.567
Jun.	.996	.998	990.737
Jul.	.997	.998	990.965
Aug.	.996	.998	990.553
Sep.	.995	.997	990.016

Table 1: Evaporation constants for Cahora Bassa

Active storage, in mln.m^3, should be not less than a minimum volume required to maintain fish population life and not greater than the maximum

$$1344 \leq s_t^c \leq M^c. \tag{3}$$

We assume that four generating units are operational (out of five). The maximum water through each turbine is 1212.19 mln.m^3/month. Therefore

$$0 \leq u_{1t}^c \leq 4848.76 \tag{4}$$

There are also constraints on the volume u_{2t}^c of spilled water. Given the structure of the dam, release through spillgates will depend on the level of the reservoir. We provide the maximum release as a function of the level l_t, measured in meters above sea level:

$$0 \leq u_{2t}^c \leq \begin{cases} 36761, & \text{if } l_t \geq 326 \\ 3423.822\sqrt{l_t - 210}, & \text{if } 320 \leq l_t < 326 \\ 3338.186\sqrt{l_t - 210}, & \text{if } l_t < 320 \end{cases} \tag{5}$$

We provide a relation between storage and level, which we obtain fitting by least squares a curve

$$l_t = l_t(s_t^c) = \alpha(1 - \exp(\tau s_t^c)) + \gamma.$$

The estimates are

$$\alpha = 48.83,$$

$$\tau = -0.0000183393,$$

$$\gamma = 296.11.$$

Obviously, it is important to provide a relation for the energy produced. We first relate head h_t, level l_t and tailrace r_t, in meters:

$$h_t = l_t - r_t.$$

Using available reservoir design data, we fit a model, by least squares, relating tailrace and release

$$r_t = \beta \log(u_t^c) + \delta,$$

with estimates $\beta = 10.3816$, $\delta = 113.8537$. Finally, the energy produced in GWh/month is

$$E_t^c = \eta u_{1t}^c h_t \tag{6}$$

with

$$\eta = 0.002725 \times .85 \times .778 = .00180204.$$

We may then build the following table showing consequences of various release policies u_1^c, u_2^c, as a function of the inflow i^c and the available amount of water in Cahora Bassa $h(i^c) = d_1^c s^c + d_2^c i^c + d_3^c$, where, for simplicity, we drop subindices t.

We use the notation

$$
\begin{aligned}
h_1 &= \frac{d_2^c u_1^c - d_1^c s^c - d_3^c}{d_2^c} \\
h_2 &= \frac{d_2^c u^c - d_1^c s^c - d_3^c}{d_2^c} \\
h_3 &= \frac{M^c + d_2^c u^c - d_1^c s^c - d_3^c}{d_2^c}
\end{aligned}
$$

and consider only the case $0 \leq h_1$, in which if $i^c = 0$, we would not be able to meet the operator's demands concerning water through turbines, that is, $u_1^c \geq h(0)/d_2^c$.

Inflow	Fin.Stor.	Energy
$i^c \leq h_1$	0	$\eta \frac{h(i^c)}{d_2^c}[l(s^c) - \beta \log(\frac{h(i^c)}{d_2^c}) - \delta]$
$h_1 < i^c \leq h_2$	0	$\eta u_1^c[l(s^c) - \beta \log(\frac{h(i^c)}{d_2^c}) - \delta]$
$h_2 < i^c \leq h_3$	$d_2^c i^c - d_2^c h_2$	$\eta u_1^c[l(s^c) - \beta \log(u^c) - \delta]$
$i^c > h_3$	M^c	$\eta u_1^c[l(s^c) - \beta \log(s^c \frac{d_1^c}{d_2^c} + i^c + \frac{d_3^c}{d_2^c} - \frac{M^c}{d_2^c}) - \delta]$

Table 2: Consequences for Cahora Bassa policies. Case $h_1 \geq 0$.

Similarly, when the inflow to Kariba is i^k, the available water in Kariba reservoir will be $g(i^k) = d_1^k s^k + d_2^k i^k + d_3^k$. Then, if

$$
\begin{aligned}
g_1 &= \frac{d_2^k u_1^k - d_1^k s^k - d_3^k}{d_2^k}, \\
g_2 &= \frac{d_2^k u^k - d_1^k s^k - d_3^k}{d_2^k},
\end{aligned}
$$

$$
\begin{aligned}
g_3 &= \frac{M^k + d_2^k u^k - d_1^k s^k - d_3^k}{d_2^k}, \\
g_4 &= g_1 + \frac{M^k}{d_2^k},
\end{aligned}
$$

where d_1^k, d_2^k, d_3^k are evaporation constants for Kariba, we have the following table of consequences (see RS for further details):

Inflow	Spill.	Total Rel.	Fin.Stor.	Inflow	Def.
$i^k \leq g_1$	0	$g(i^k)/d_2^k$	0		
$g_1 < i^k \leq g_2$	$i^k - g_1$	$g(i^k)/d_2^k$	0	$i^k \leq o_1$	0
$g_2 < i^k \leq g_3$	u_2^k	$u_1^k + u_2^k$	$d_2^k(i^k - g_2)$		
$i^k > g_3$	$i^k - g_4$	$(g(i^k) - M^k)/d_2^k$	M^k	$i^k > o_1$	1

Table 3: Consequences for Kariba policies. Case $g_1 \geq 0$.

Again, we have included only the case $0 \leq g_1$, in which, if there was no inflow to Kariba, we would not be able to meet the operator's demands concerning water through turbines. o_1 is a minimum inflow level not leading to energy deficit, as a consequence of the energy produced being a nondecreasing function of the inflow.

3.3 Forecasting models

The main source of uncertainty in reservoir management is associated with the inflow process: monthly releases should take into account the inflow in the corresponding and later months, which are uncertain. In order to solve the forecasting problem, we employ Bayesian Dynamic Models (DM). West and Harrison (1989) and West (1995) describe these models comprehensively.

Given the location of both reservoirs, we consider a forecasting model for the inflows to Kariba, which we only outline (see RS for details), and a forecasting model for the inflows to Cahora Bassa. This one will depend on those inflows not coming from Kariba, which we call incremental inflows, and on the releases from Kariba. This accounts for interreservoir dynamics.

3.3.1 Inflows to Kariba Lake

After log-transformation, we modeled the Kariba inflow time series with a level term, a term representing seasonal (annual) variation and a low coefficient, first order autoregressive term to improve short term forecasts. We only retained the first harmonic in the seasonal part.

Figure 2 here

Figure 2: Logarithm of Kariba inflows, in mln.m^3.

Consequently, we ended up working with the following Dynamic Linear Model (DLM):

Observation equation.

$$y_t^k = z_t^{1k} + z_t^{2k} + z_t^{4k} + v_t^k, \quad v_t^k \sim N(0, v^k)$$

where $y_t^k = \log(i_t^k)$ is the logarithm of the inflow to Kariba; z_t^{1k} designates the level of the series; z_t^{2k} and z_t^{3k} refer to the seasonal term, see below; z_t^{4k} refers to the autoregressive term; and v_t^k designates a Gaussian error term of constant, but unknown, variance v^k.

System equation.

$$\begin{aligned} z_t^{1k} &= z_{t-1}^{1k} + w_t^{1k} \\ z_t^{2k} &= \cos(\pi/6) z_{t-1}^{2k} + \sin(\pi/6) z_{t-1}^{3k} + w_t^{2k} \\ z_t^{3k} &= -\sin(\pi/6) z_{t-1}^{2k} + \cos(\pi/6) z_{t-1}^{3k} + w_t^{3k} \\ z_t^{4k} &= .4 z_{t-1}^{4k} + w_t^{4k} \end{aligned}$$

with $\mathbf{w}_t^k = (w_t^{1k}, w_t^{2k}, w_t^{3k}, w_t^{4k})$ an error term such that

$$\mathbf{w}_t^k \sim N\left(\mathbf{0}, \begin{pmatrix} v^k W_t^{*k} & 0 \\ 0 & \sigma^{k2} \end{pmatrix}\right),$$

where σ^{k2} is the autoregressive variance; and W_t^{*k}, the variance matrix (up to v^k) of the first three terms. This matrix was defined using discounting, with a discount factor of .8 for the level and a discount factor .95 for the seasonal part.

Prior information.

$$\mathbf{z}_0^k|\phi^k \sim N(\mathbf{m}_0^k, v^k C^{*k})$$

$$\phi^k \sim Gamma(n_0^k/2, d_0^k/2)$$

with $\mathbf{z}_0^k = (z_0^{1k}, z_0^{2k}, z_0^{3k}, z_0^{4k})$ and $\phi^k = \frac{1}{v^k}$.

Prior parameters were specified judgementally, with $\mathbf{m}_0^k$ as (7.8,-1.02,.33,0),

$$C^{k*} = \begin{pmatrix} .02 & 0 & 0 & 0 \\ 0 & .002 & .0007 & 0 \\ 0 & .0007 & .003 & 0 \\ 0 & 0 & 0 & .1 \end{pmatrix},$$

$n_0^k = 10$ and $d_0^k = .8$, and sensitivity thoroughly checked.

3.3.2 Dynamic regression Kariba-Cahora Bassa

Exploratory data analyses suggested regressing dynamically the inflows to Kariba and Cahora Bassa, after a log-transformation.

Figure 3 here

Figure 3: Difference of logarithms of inflows to Cahora and Kariba.

For that, we define $y_t^c = \log(i_t^c)$ and $z_t = y_t^c - y_t^k$. We try to forecast z_t plotted above, again through a level and a seasonal part. In this case, since the expert had little feeling about this variable, we based the assessments on the data as reflected in Table 4. We checked them afterwards via sensitivity analysis.

	Lev	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep
Mean	.85	1.64	1.59	1.06	.91	.85	.41	-.16	-.04	.36	.86	1.23	1.46
Effect		-.79	-.74	-.21	-.06	-.00	.43	1.01	.89	.48	-.01	-.38	-.61
Maxdev	.33	.35	.38	1.06	1.11	.98	1.13	1.01	.85	.68	.69	.49	.39
Stdev	.16	.00	.02	.36	.39	.32	.39	.33	.25	.17	.17	.08	.03

Table 4: Assessments for level and seasonal effects.

Variances were deduced accordingly, assuming independence between level and monthly effects. For example, for the level, the maximum deviation from the expected value is about .33 and we assume a standard deviation of .165.

The Fourier decomposition of the seasonal part was

Harmonic	0	1	2	3	4	5	6
a_i	0.	-.779	.056	-.122	.033	-.003	.017
b_i	-	-.103	.143	-.091	-.048	-.012	-

Table 5: Fourier decomposition of seasonal effects.

with covariance matrices deduced from those of the effects. For example, for the first harmonic

$$C_1^* = \begin{pmatrix} .017 & .005 \\ .005 & .015 \end{pmatrix}.$$

Relevance of harmonics was assessed with methods described in West and Harrison (1989) with results:

Harmonic	1	2	3	4	5	6
Statistic	-5.77	-.15	.49	-.28	-.00	.00

Table 6: Relevance of harmonics.

As a consequence, only the first harmonic was retained initially. However, posterior analyses suggested the inclusion of the second harmonic as beneficial, together with a low coefficient, first order autoregressive term to improve short term forecasts. Consequently, we ended up working with the following DLM:

Observation equation.

$$z_t = s_t^1 + s_t^2 + s_t^4 + s_t^6 + v_t^c, \quad v_t^c \sim N(0, v^c)$$

where s_t^1 designates the level of the series; s_t^2 and s_t^3 refer to first harmonic of the seasonal term; s_t^4 and s_t^5 refer to the second harmonic and s_t^6 refers to the autoregressive term; and v_t^c designates a Gaussian error term of constant, but unknown, variance v^c.

System equation.

$$
\begin{aligned}
s_t^1 &= s_{t-1}^1 + w_t^{1c} \\
s_t^2 &= \cos(\pi/6)s_{t-1}^2 + \sin(\pi/6)s_{t-1}^3 + w_t^{2c} \\
s_t^3 &= -\sin(\pi/6)s_{t-1}^2 + \cos(\pi/6)s_{t-1}^3 + w_t^{3c} \\
s_t^4 &= \cos(\pi/3)s_{t-1}^4 + \sin(\pi/3)s_{t-1}^5 + w_t^{4c} \\
s_t^5 &= -\sin(\pi/3)s_{t-1}^4 + \cos(\pi/3)s_{t-1}^5 + w_t^{5c} \\
s_t^6 &= .4s_{t-1}^6 + w_t^{6c}
\end{aligned}
$$

with $\mathbf{w}_t^c = (w_t^{1c}, w_t^{2c}, ..., w_t^{6c})$ an error term such that

$$\mathbf{w}_t^c \sim N\left(\mathbf{0}, \begin{pmatrix} v^c W_t^{*c} & 0 \\ 0 & \sigma^{c2} \end{pmatrix}\right),$$

with σ^{c2} the autoregressive variance; and W_t^{*c}, the variance matrix (up to term v^c) of the first five terms. This matrix was defined using discounting, with a level discount factor δ_1^c and a seasonal part discount factor δ_2^c.

Prior information.

$$\mathbf{s}_0|\phi^c \sim N(\mathbf{m}_0^c, v^c C^{*c})$$

$$\phi^c \sim Gamma(n_0^c/2, d_0^c/2)$$

with $\mathbf{s}_0 = (s_0^1, s_0^2, ..., s_0^6)$ and $\phi^c = \frac{1}{v^c}$.

The assessment of the gamma parameters was more complicated, and we based it on a guess for the expected variance and graphical representations of the corresponding densities. As a result of the assessments, $\mathbf{m}_0^c$ was specified as (.850,-.779,-.103,.056,.144,0),

$$C^{*c} = \begin{pmatrix} .09 & 0 & 0 & 0 & 0 & 0 \\ 0 & .017 & .005 & 0 & 0 & 0 \\ 0 & .005 & .015 & 0 & 0 & 0 \\ 0 & 0 & 0 & .02 & -.002 & 0 \\ 0 & 0 & 0 & -.002 & .022 & 0 \\ 0 & 0 & 0 & 0 & 0 & .1 \end{pmatrix},$$

$n_0^c = 8$ and $d_0^c = 1.2$.

Extensive sensitivity analyses were conducted with respect to: δ_1^c and δ_2^c; the gamma parameters n_0^c and d_0^c; the autoregressive coefficient and variance. We studied the effect of changes in the initial estimates of these parameters on the behaviour of predictive variances, mean absolute error of one step ahead forecast errors and their autocorrelation functions. The model seemed fairly robust and we selected the following discount parameters: $\delta_1^c = .95$ and $\delta_2^c = .95$.

3.3.3 Inflows to Cahora Bassa

The model we propose now for Cahora Bassa depends on the release from Kariba and the incremental inflow (inc_t) to Cahora Bassa from the tributaries and basin between Kariba Gorge and inlet to Cahora Bassa. Taking into account that water travelling time is less than a month, we use the following relation:

$$i_t^c = u_t^k + inc_t.$$

Given that data about releases are not available (and they will actually depend on releases of our approach) we shall estimate incremental inflows as follows. First, if there was no reservoir, we would have:

$$i_t^c = i_t^k + inc_t.$$

The second relation describes a dynamic regression between inflows to Cahora and Kariba, and reflects the physical relation that inflow is related with basin size:

$$i_t^c = \beta_t i_t^k.$$

Simple computations suggest modelling incremental inflows by

$$inc_t = (\beta_t - 1) i_t^k,$$

and inflows to Cahora by

$$i_t^c = u_t^k + (\beta_t - 1) i_t^k.$$

Observe that we no longer have a dynamic linear model, since both the regression weights and regressors are subject to uncertainty. Besides, the distribution of u_t^k is not standard. Forecasting i_t^c is easily done by simulation, since we know or can easily sample the distributions involved. That of i_t^k is obtained from the model in Section 3.3.1; u_t^k is obtained from Section 3.3.1 and using the table from consequences for Kariba in Section 3.2; the distribution on β_t comes from the inversion of the transformation in Section 3.3.2 and the model there. After a sufficient number of stages, it will follow a lognormal distribution.

3.4 Utility functions

Following the description in Section 1.2, the utility function used in our approach takes into account the consequences of interest, described in Section 3.2, and deviations to a reference trajectory. We describe first how we compute reference trajectories and then the full preference model.

3.4.1 Computation of the reference trajectory

Here we describe the computation of the reference trajectory for Cahora Bassa. A similar procedure was applied in RS to obtain one for Kariba. Essentially, we use deterministic versions of our problems, where inflows are considered known and fixed at their predictive expected values. The dynamics are the same of Section 3.2. We set the planning period equal to one year.

For the deterministic problem, we need an initial volume. We adopt

$$s_0^c = .7M^c = 42000. \tag{7}$$

The objective function includes a term relating to deviation from the initial state, a term relating to the main objective in the stochastic problem, which is energy production, and a term which deals with separation from the states allowing for fishing (storage between 41500 and 51600), defined by the function

$$g(s^c) = \begin{cases} 0, & 41500 \leq s^c \leq 51600 \\ \frac{1}{2141400000}(s^{c2} - 93100 s^c) + 1, & \text{otherwise} \end{cases}$$

Then, the objective function is

$$f(u^c) = \rho_0 (s_{12}^c - s_0^c)^2 - \rho_1 \sum_{t=1}^{12} \eta u_{1t}^c (l_t - r_t) + \rho_2 \sum_{t=1}^{12} g(s_t^c),$$

with ρ_0, ρ_1, ρ_2 positive numbers chosen to give similar weight to the three components and improve numerical stability.

The optimisation problem is $\min f(u^c)$ s.t. (1) – (7). We solved it by dynamic programming, with several reformulations to simplify the solution. Each subproblem at each stage was solved with a modified version of OPQSQP (NOC, 1990). A multistart strategy was adopted to avoid bad local optima. For every month t, a smooth function was adjusted as an approximation to the optimal value function.

The reference trajectory was obtained as the optimal solution of the above optimisation problem and is described in Table 7. An additional constraint $u_{2t}^c \leq 18000$ was introduced to achieve a more homogeneous performance.

Month	Initial	Rel. Tur.	Spill	Energy	Final
October	42000	4848	0	1052	42256
November	42256	4848	0	1053	41420
December	41420	4848	0	1049	43293
January	43293	4848	0	1047	48258
February	48258	4848	0	1063	55948
March	55948	4848	18000	956	45969
April	45969	4848	18000	922	32458
May	32458	4848	0	1013	35660
June	35666	4848	0	1026	38037
July	38037	4848	0	1036	40065
August	40065	4848	0	1044	41457
September	41457	4848	0	1048	42179

Table 7: Reference trajectory for Cahora Bassa.

It corresponds to an excellent reservoir performance, with high energy output and fairly homogeneous releases.

3.4.2 The expression of the utility function

As described in RS, for Kariba we used an additive decomposition for the utility function, because of good first order approximation. Therefore, we include a term relating to deficit and a term relating to spills. The last term is a penalty to deviations from the reference state. The general form is:

$$F_K(u_1^k, u_2^k) = F_K(k, u_2^k, s^k) = \lambda f_1(k) + (1-\lambda) f_2(u_2^k) + \rho_K (s^k - s^{k*})^2$$

where k represents the existence (1) or not (0) of deficit; s^k is the final state of the reservoir; s^{k*} is the reference state; u_2^k is the volume of water spilled; λ and ρ_K are weights; and f_1 and f_2 are component utility functions. For f_1, given that k may only attain two values 0 (no deficit) and 1 (deficit), and 0 is better, we choose

$$f_1(k) = 1 - k.$$

For f_2, given that current management is risk averse to large releases and small releases are preferred to large ones, we chose a (risk averse) nondecreasing exponential utility function

$$f_2(u_2^k) = -.07171 + 1.08365 \exp(-.0001415 u_2^k).$$

We used standard assessment methods, see e.g. French (1986), for eliciting the parameters, with $\lambda = .75$. We chose $\rho_K = -10^{-10}$ to appropriately scale penalties with other component utilitites.

For Cahora Bassa, we modelled

$$F_C(u_1^c, u_2^c) = F_C(E^c, s^c) = f_3(E^c) + \rho_C(s^c - s^{c*})^2,$$

where E^c is the energy produced, s^c and s^{c*} are the storage and reference storage at Cahora Bassa. Using standard utility assessment techniques, we used a convex-concave utility function with 765 GWh/month as inflection point:

$$f_3(E^c) = \begin{cases} 0.0038(-1 + \exp(0.0066E^c)), & \text{if } E^c \leq 765 \\ 1.003473 - .4034728 \exp(-.02377571(E^c - 765)), & \text{otherwise} \end{cases}$$

For the same reasons above, we used $\rho_C = -10^{-10}$.

Finally, both utility functions are composed so that

$$F(u_1^k, u_2^k, u_1^c, u_2^c) = \mu F_K(u_1^k, u_2^k) + (1 - \mu) F_C(u_1^c, u_2^c)$$

and the expected utility is

$$\Psi(u_1^c, u_2^c, u_1^k, u_2^k) = \int F(u_1^c, u_2^c, u_1^k, u_2^k)\, dH(i^k, i^c|D),$$

where $H(i^k, i^c|D)$ is the predictive distribution for the inflows to the reservoirs, obtained from the forecasting model in Section 3.3. As initial value for μ we used 0.5. This, and other parameters, were checked via sensitivity analysis.

3.5 Expected utility maximization

At every time step, the expected utility function had to be maximized with respect to control variables and subject to constraints on the controls (releases from the reservoir): the amount of water released has to be nonnegative and the amounts of water released for energy production and flow control are limited by the capacity of the turbines and the spillgates. The optimization problem is thus given as:

$$\begin{aligned} & \max \;\; \Psi(u_1^c, u_2^c, u_1^k, u_2^k) \\ s.t. \;\; & 0 \leq u_1^k \leq m^k \\ & 0 \leq u_2^k \\ & 0 \leq u_1^c \leq m^c \end{aligned}$$

with the constraints imposed on u_2^c due to the structure of the Cahora Bassa dam, as in (5). m^k and m^c designate the maximum releases through Kariba and Cahora Bassa turbines, respectively.

Note, though, that we do not have an explicit expression for the expected utility. The complications from the calculation are reflected in the following influence diagram describing the decision making problem at each stage.

Figure 4 here

Figure 4: Decision problem at each stage

However, for each $(u_1^k, u_2^k, u_1^c, u_2^c)$, the expected utility may be easily computed by simulation:

For $(u_1^k, u_2^k, u_1^c, u_2^c)$,

1. Generate $(i_k^1, ..., i_k^N)$, from the predictive distribution in 3.3.1

2. For each i_j^k, and given $(u_1^k, u_2^k, u_1^c, u_2^c)$

   -Compute spill, total release, final storage, deficit, $F_K(u_1^k, u_2^k)_j$ (with appropriate tables, like Table 3)

   -From the predictive distribution in 3.3.1,3.3.2 and 3.3.3 and release from Kariba, generate i_j^c

   -Compute energy, final storage, $F_C(u_1^c, u_2^c)_j$ (with appropriate tables, like Table 2).

3. Approximate $\Psi(u_1^c, u_2^c, u_1^k, u_2^k)$ by

$$\hat{\Psi}_N(u_1^c, u_2^c, u_1^k, u_2^k) = \frac{1}{N}\sum_{j=1}^{N}\left(\mu F_K(u_1^k, u_2^k)_j + (1-\mu)F_C(u_1^c, u_2^c)_j\right)$$

Consequently, at each stage we solve the problem

$$\max \ \ \hat{\Psi}_N(u_1^c, u_2^c, u_1^k, u_2^k)$$

with the constraints above.

We solved those problems, with Nelder Mead algorithm, as implemented in IMSL (1989), the main reason being the need for a robust method, given the changes in objective function at different stages, and the need of a method requiring only function evaluations. To avoid convergence to bad local optima, we used a guided multistart strategy.

3.6 Performance

The proposed approach was tested with monthly inflow data from October 1930 to September 1965. Simulations included a warming up period for the forecasting algorithm, followed by a period in which both forecasting and optimisation took place. The first issue we analysed was the impact of key parameters in the performance of the reservoir, mainly μ, ρ_K and ρ_C. Sensitivity analysis of other parameters has been described in RS.

First, since spills from Kariba were fairly irregular, ρ_K was modified to -10^{-9}, and, for homogeneity, a similar change was made for ρ_C. In spite of improvements, we decided to include upper bounds for announced spills from both reservoirs. Effectively, we introduced maximum announced spills of 4000 and 8000 mln.m^3, for

Kariba and Cahora Bassa respectively, reflecting the desire to control more tightly Kariba, and the relatively less dangerous downstream effects of Cahora spills. Moreover, since at high inflow levels reservoirs tended to be too full, a rule was introduced which increased ρ_K and ρ_C to -5×10^{-9} when reservoirs were at 90% of their maximum storage. μ was initially fixed at .5, and varied in the range [0,1]. The behaviour was fairly stable in the range [.4,.6], whereas for more extreme values a more irregular behaviour was appreciated. As a consequence, our final simulations were made with our initial value of $\mu = .5$. The figures provide the behaviour of both reservoirs with our policies, showing releases through turbines, spilled water, energy produced and storage.

Figure 5 here

Figure 5: Simulated performance of Kariba: releases, storage and energy output.

Figure 6 here

Figure 6: Simulated performance of Cahora: releases, storage and energy output.

As Kariba is concerned, note that the energy output is well above the target set of 750 GWh/month, which is much higher than the current target of 600 GWh/month, hence suggesting potential for a much more efficient operation. Regarding releases, they are always below the set up level of 4000 mln.m^3, except for two periods: one of three months, in which release were 19451, 11331 and 4491, respectively; another of two months, with spills of 5392 and 7108, respectively. Note, though, that the inflows in those months were 25486, 27690 and 14281 (enough to fill up the reservoir!!!), and 12295, 17475, 15899, 10637, therefore corresponding to a fairly acceptable regulation of the inflow. Globally, the spills are fairly homogeneous and much smaller than those under current management. Also, except for those five months in which the reservoir was at maximum retention level, the operation was under safe conditions. Releases through turbines tend to be close to the maximum of 3820 mln.m^3.

A similar pattern emerges for Cahora Bassa. Releases through turbines tend to be close to the maximum of 4848.76. Spills are smaller than 8000, and fairly homogeneous, except for six months with spills of 9093 (with inflows the previous four months equaling 56000), 29302 and 13539 (with inflows the previous four months of 81000) and 17506, 15116, 9794 (with inflows the previous six months of 106000). Again, we operate at fairly safe levels regulating inflows satisfactorily. The energy output ranges between 895 GWh/month and 1066 GWh/month. The operation of both reservoirs is fairly well balanced, both in economic and safety terms.

We tested several scenarios. First, we studied performance of the reservoirs starting from very low storages. Initially, the energy production was not very high, in an attempt to fill up the reservoir progressively, until high enough storages were achieved, after which the performance was good enough. Similarly, we tested the operation of the reservoir for initial storages close to their maxima: again reservoir gradually reached safer (lower) levels. In this case, adaptation was much faster than that described in RS, given the rule for high storages introduced.

Finally, we simulated the reservoir with very low inflows. Since real inflows available were not low, we generated inflows artificially by dividing real ones by 4. Under these extreme conditions the performance was still acceptable, in the sense of there being only releases for energy production and only so as to satisfy energy targets. Obviously, those settings are too high for very low inflows suggesting the need of changing targets under these conditions or reformulation of objectives and preferences.

The typical shape of the release rules obtained as the result of the computations is: for low values of storage, the amount of water released for energy production is decreasing as the storage increases, since smaller amounts of water are required to achieve given energy production target and we want to remain close to the reference state; at high storage levels, the operating policy tends to release as much water as possible to generate electricity, minimizing spills. For very high storage levels, the mechanism controlling deviation of the current state from the reference values 'turns on' and excessive water is spilled in order to secure that the reference trajectory is followed.

Figure 7 here

Figure 7: Typical shape of policies

4 Discussion

Various reviews (e.g., Yakowitz, 1982; Yeh, 1985) of reservoir operations describe multireservoir operation problems under uncertainty as extremely complex. In our case, there were additional complications due to the presence of multiple objectives. We have shown that our approach may succesfully cope with this kind of problems.

Key issues are the definition of flexible policies; the use of Bayesian forecasting models; a careful modelling of preferences, which include a term reflecting deviation from a reference trajectory; a heuristic providing policies of approximate maximum expected utility; and, thorough checking of our policies through sensitivity analyses, to provide additional modelling insights.

We would like to stress that a basic principle underlying our approach is that of *management by exception*, West and Harrison (1989). The central idea is that we use a set of models to process information, make predictions and decisions, unless exceptional circumstances arise, in which subjective intervention is required. This may be reflected in various ways. For example, the forecasting model could require interventions if sudden rainfall suggests an increase in the inflow level. It is also reflected in the use of predictive expected inflows in the definition of the reference trajectory, with obvious differences from wet to dry years. Finally, it is also reflected in the rule introduced to handle cases in which reservoir storages are very high. Obviously, other types of interventions would be permitted, making our approach truly interactive.

This case study, and our previous RS, suggest the enormous potential of our approach in handling reservoir operation problems, and, more generally, multistage decision problems under uncertainty. However, its application is far from simple. We are currently building Bayres (Ríos Insua, Bielza, Martín, Salewicz (1996)), a decision support system for reservoir operations that would support all the phases of our approach. On a more theoretical stand, we are evaluating the goodness of our heuristic in general multistage decision problems. Other applications of Bayesian methods in water resources problems are described in Berger and Ríos Insua (1996).

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Figure 1:

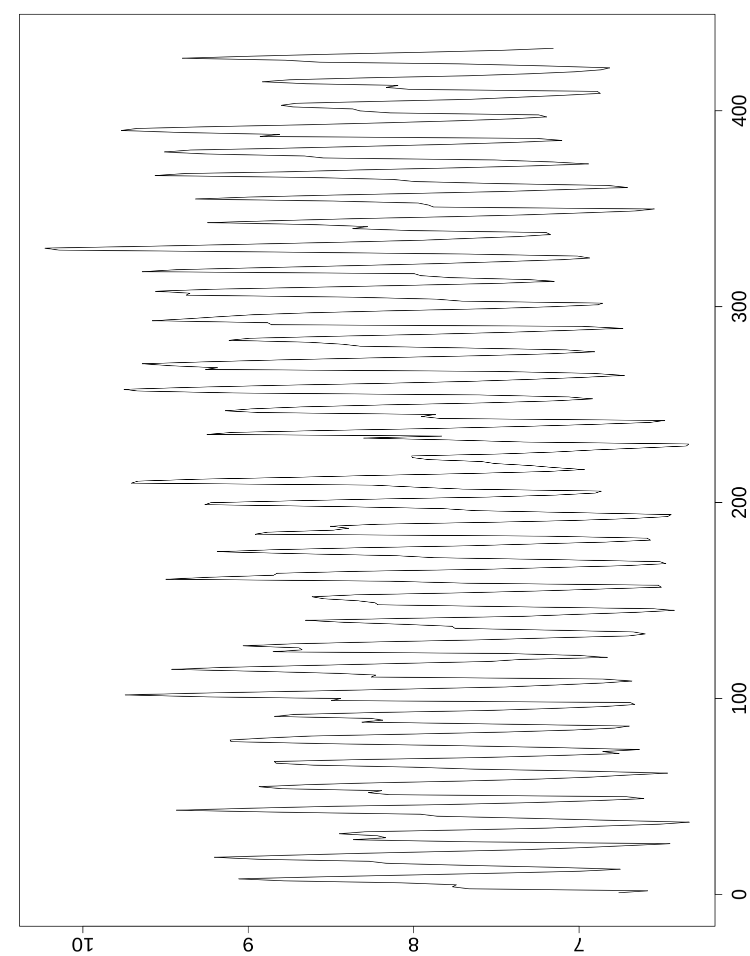

Figure 2:

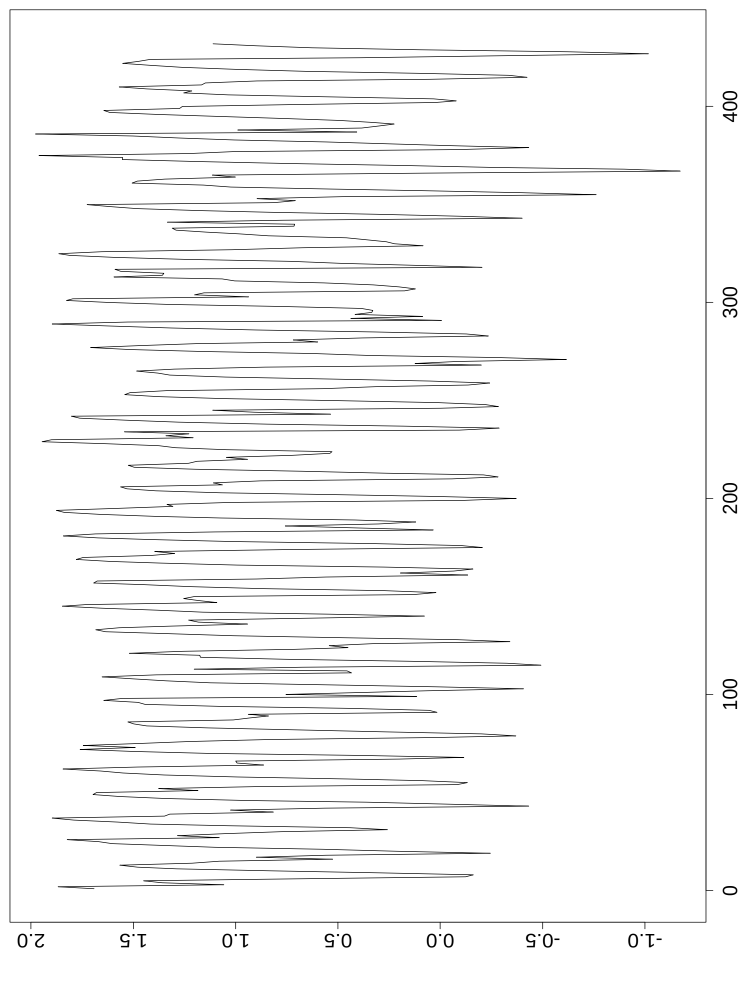

Figure 3:

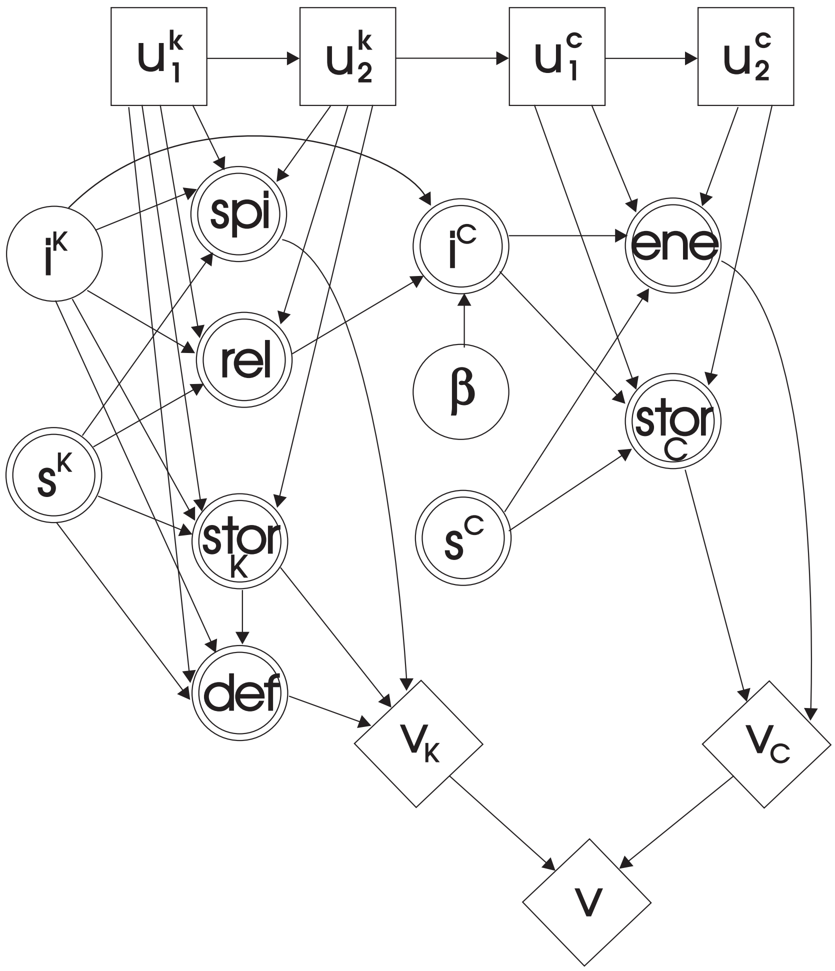

Figure 4:

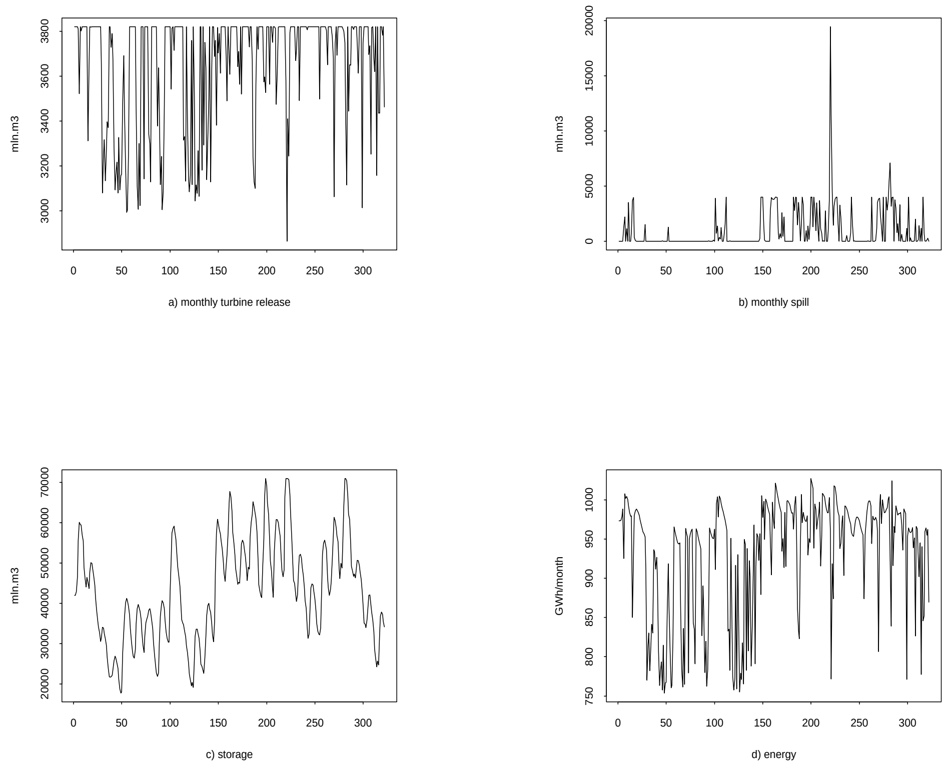
mln.m3
3000
3200
3400
3600
3800
0
50
100
150
200
250
300
a) monthly turbine release
mln.m3
0
5000
10000
15000
20000
b) monthly spill
mln.m3
20000
30000
40000
50000
60000
70000
c) storage
GWh/month
750
800
850
900
950
1000
d) energy

Figure 5:

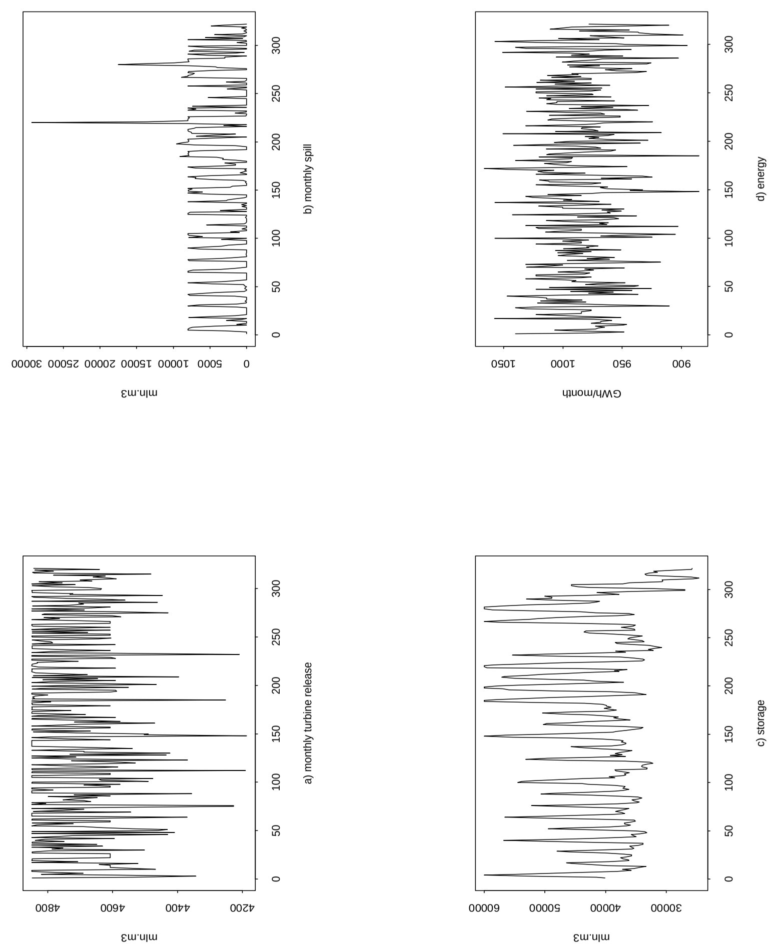
a) monthly turbine release
mln.m3
b) monthly spill
mln.m3
c) storage
mln.m3
d) energy
GWh/month

Figure 6:

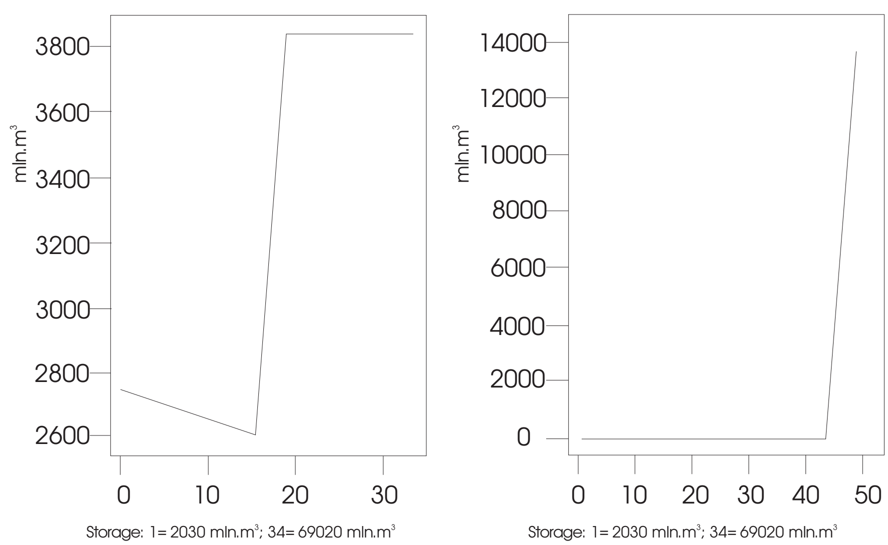
3800
3600
3400
3200
3000
2800
2600
mln.m³
0
10
20
30
Storage: 1= 2030 mln.m³; 34= 69020 mln.m³
14000
12000
10000
8000
6000
4000
2000
0
mln.m³
0
10
20
30
40
50
Storage: 1= 2030 mln.m³; 34= 69020 mln.m³

Figure 7: